Graphing Hyperbolas

The standard equation for a hyperbola can take one of two forms:

$\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$ with vertices at $(h \pm a, k)$ or

$\dfrac{(x-k)^2}{b^2} - \dfrac{(y-h)^2}{a^2} = 1$ with vertices at $(h, k \pm b)$.

There is a problem entering this equation in the calculator graphing list for two reasons:

a) it is not a function, and only functions can be entered in the Y= list locations.

b) the functions entered in the Y= list locations must be in terms of x, not y.

To draw a graph of a hyperbola, consider the "top" and "bottom" halves of the hyperbola as two different parts of the graph because each individual is a function. Solve the equation of the hyperbola for y and enter the two parts in two locations of the Y= list.

Example

Graph a hyperbola in rectangular mode. Solve the equation for y to put it in the standard form.

Graph the hyperbola $x^2 + 2x - y^2 - 6y + 3 = 0$

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Set the zoom to the decimal window: | ZOOM | | A | (| ENTER | | ALPHA | | ▼ |) | 7 |

Step & Key Operation	Display	Notes

1 Solve the equation for y completing the square.

Enter
$Y1 = \sqrt{x^2 + 2x + 12}$
$Y2 = Y1 - 3$
$Y3 = -Y1 - 3$

| Y= | | 2nd F | | √ | | X/θ/T/n | | X² | | + | | 2 |

| X/θ/T/n | | + | | 1 | | 2 | | ENTER |

| 2nd F | | VARS | | A | | ENTER | | 1 | | — | | 3 | | ENTER |

| (-) | | 2nd F | | VARS | | A | | ENTER | | 1 | | — | | 3 |

Display:
```
Y1■√X²+2X+12
Y2■Y1−3
Y3■-Y1−3
Y4=
Y5=
Y6=
```

Notes:
$x^2 + 2x - y^2 - 6y = -3$
$x^2 + 2x - (y^2 + 6y + 9) = -3 -9$
$x^2 + 2x - (y+3)^2 = -12$
$(y+3)^2 = x^2 + 2x + 12$
$y + 3 = \pm\sqrt{x^2 + 2x + 12}$
$y = \pm\sqrt{x^2 + 2x + 12} - 3$

2 Turn off Y1 so that it will not graph.

| ▲ | | ▲ | | ◄ | | ENTER |

Display:
```
Y1=√X²+2X+12
Y2■Y1−3
Y3■-Y1−3
Y4=
Y5=
Y6=
```

SHARP

Step & Key Operation	*Display*	*Notes*

3 View the graph.

GRAPH
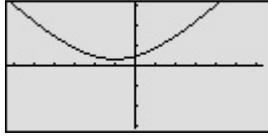
4 Zoom out the screen.

ZOOM A 4

• •
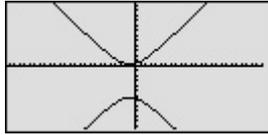
Graphing hyperbolas can be performed easily on the calculator display.

SHARP