# Graphing Rational Functions

A rational function $f(x)$ is defined as the quotient $\dfrac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are two polynomial functions such that $q(x) \neq 0$. The domain of any rational function consists of all values of x such that the denominator $q(x)$ is not zero.

A rational function consists of branches separated by vertical asymptotes, and the values of x that make the denominator $q(x) = 0$ but do not make the numerator $p(x) = 0$ are where the vertical asymptotes occur. It also has horizontal asymptotes, lines of the form $y = k$ (k, a constant) such that the function gets arbitrarily close to, but does not cross, the horizontal asymptote when $|x|$ is large.

The x intercepts of a rational function $f(x)$, if there are any, occur at the x-values that make the numerator $p(x)$, but <u>not</u> the denominator $q(x)$, zero. The y-intercept occurs at $f(0)$.

Example

Graph the rational function and check several points as indicated below.

1. Graph $f(x) = \dfrac{x-1}{x^2-1}$.

2. Find the domain of $f(x)$, and the vertical asymptote of $f(x)$.

3. Find the x- and y-intercepts of $f(x)$.

4. Estimate the horizontal asymptote of $f(x)$.


Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Set the zoom to the decimal window: `ZOOM` `A` (`ENTER` `ALPHA` `▼`) `7`

Step & Key Operation | **Display** | **Notes**

1-1 Enter $y = \dfrac{x-1}{x^2-1}$ for Y1.

`Y=`

`a/b` `X/θ/T/n` `—` `1` `▼` `X/θ/T/n` `X²`
`—` `1`
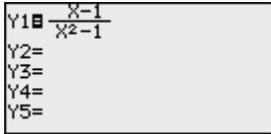
1-2 View the graph.

`GRAPH`
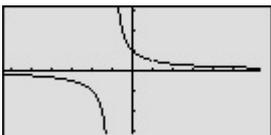
The function consists of two branches separated by the vertical asymptote.

SHARP

Step & Key Operation	Display	Notes

2 Find the domain and the vertical asymptote of $f(x)$, tracing the graph to find the hole at $x = 1$.
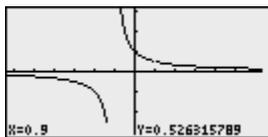
Since $f(x)$ can be written as $\dfrac{x - 1}{(x + 1)(x - 1)}$, the domain consists of all real numbers x such that $x \neq 1$ and $x \neq -1$. There is no vertical asymptote where $x = 1$ since this value of x also makes the numerator zero. Next to the coordinates $x = 0.9$, $y = 0.52$, see that the calculator does not display a value for y at $x = 1$ since 1 is not in the domain of this rational function.

TRACE ▶ (repeatedly)

▶

▶
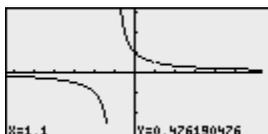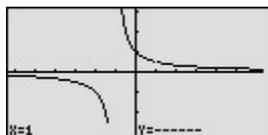
3 Find the x- and y-intercepts of $f(x)$.

2nd F CALC 6
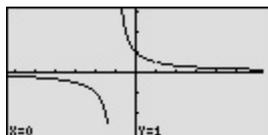
The y-intercept is at (0 ,1). Notice that there are no x-intercepts for the graph of $f(x)$.

4 Estimate the horizontal asymptote of $f(x)$.

The line $y = 0$ is very likely a horizontal asymptote of $f(x)$.

The graphing feature of the EL-9900 can create the branches of a rational function separated by a vertical asymptote. The calculator allows the points of intersection to be obtained easily.

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