Solving Absolute Value Inequalities

To solve an inequality means to find all values that make the inequality true. Absolute value inequalities are of the form $|f(x)| < k$, $|f(x)| \leq k$, $|f(x)| > k$, or $|f(x)| \geq k$. The graphical solution to an absolute value inequality is found using the same methods as for normal inequalities. The first method involves rewriting the inequality so that the right-hand side of the inequality is 0 and the left-hand side is a function of x. The second method involves graphing each side of the inequality as an individual function.

Example

Solve absolute value inequalities in two methods.

1. Solve $|20 - \frac{6x}{5}| < 8$ by rewriting the inequality so that the right-hand side of the inequality is zero.

2. Solve $|3.5x + 4| > 10$ by shading the solution region.

Before Starting   There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Set viewing window to "-5< x <50," and "-10< y <10".

| WINDOW | (–) | 5 | ENTER | 5 | 0 | ENTER |

Step & Key Operation	Display	Notes

1-1   Rewrite the equation.

$|20 - \frac{6x}{5}| < 8$

$\rightarrow |20 - \frac{6x}{5}| - 8 < 0.$

1-2   Enter $y = |20 - \frac{6x}{5}| - 8$ for Y1.

| Y= | MATH | B | 1 | 2 | 0 | — | a/b |

| 6 | X/θ/T/n | ▶ | 5 | ▶ | ▶ |

| — | 8 |

Y1◘ $20 - \frac{6X}{5}$ -8
Y2=
Y3=
Y4=
Y5=
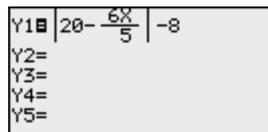
1-3   View the graph, and find the x-intercepts.

| GRAPH |

| 2nd F | CALC | 5 | → $x = 10$, $y = 0$

| 2nd F | CALC | 5 | → $x = 23.33333334$

$y = 0.00000006$ (∗ Note)
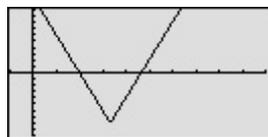
The intersections with the x-axis are (10, 0) and (23.3, 0) (∗ Note: The value of y in the x-intercepts may not appear exactly as 0 as shown in the example, due to an error caused by approximate calculation.)

1-4   Solve the inequality.

Since the graph is below the x-axis for x in between the two x-intercepts, the solution is $10 < x < 23.3$.

SHARP

Step & Key Operation	*Display*	*Notes*

2-1 Enter the function
$y = |3.5x + 4|$ for Y1.
Enter $y = 10$ for Y2.

| Y= | CL | MATH | B | 1 |

| 3 | . | 5 | X/θ/T/n | + | 4 | ENTER |

| 1 | 0 |

```
Y1=|3.5X+4|
Y2=10
Y3=
Y4=
Y5=
Y6=
```

2-2 Set up shading.

| 2nd F | DRAW | G | 1 |

| 2nd F | VARS | A | ENTER | A | 2 | ▶ |

| 2nd F | VARS | ENTER | 1 |

```
Set shade
   Y2 < Y < Y1
      < Y <
      < Y <
      < Y <
      < Y <
```

Since the inequality you are solving is Y1 > Y2, the solution is where the graph of Y2 is "on the bottom" and Y1 in "on the top."

2-3 Set viewing window to "-10 < x < 10" and "-5 < y < 50", and view the graph.

| WINDOW | (-) | 1 | 0 | ENTER | 1 | 0 |

| ENTER | ENTER | (-) | 5 | ENTER | 5 | 0 |

| ENTER | 5 | ENTER |

```
Window (Rect)
  Xmin=-10
  Xmax=10
  Xscl=1
  Ymin=-5
  Ymax=50
  Yscl=5
```

2-4 Find the points of intersection.
Solve the inequality.

| 2nd F | CALC | 2 | → $x = -4$, $y = 10$ |

| 2nd F | CALC | 2 | → $x = 1.714285714$ |

$y = 9.999999999$ (∗ Note)
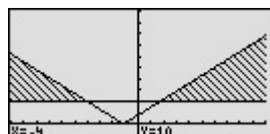
The intersections are (-4, 10) and (1.7, 10.0). The solution is all values of x such that $x < -4$ or $x > 1.7$.
(∗ Note: The value of y in the intersection of the two graphs may not appear exactly as 10 as shown in the example, due to an error caused by approximate calculation.)

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The EL-9900 shows absolute values with | |, just as written on paper, by using the Equation editor. Graphical solution methods not only offer instructive visualization of the solution process, but they can be applied to inequalities that are often difficult to solve algebraically. The Shade feature is useful to solve the inequality visually and the points of intersection can be obtained easily.

SHARP