Solving a System of Linear Equations Using Matrices

Each system of three linear equations consists of three variables. Equations in more than three variables cannot be graphed on the graphing calculator. The solution of the system of equations can be found numerically using the Matrix feature or the System solver in the Tool feature.

A system of linear equations can be expressed as AX = B (A, X and B are matrices). The solution matrix X is found by multiplying $A^{-1}B$. Note that the multiplication is "order sensitive" and the correct answer will be obtained by multiplying BA^{-1}. An inverse matrix A^{-1} is a matrix that when multiplied by A results in the identity matrix I ($A^{-1} \times A = I$). The identity matrix I is defined to be a square matrix ($n \times n$) where each position on the diagonal is 1 and all others are 0.

Example

Use matrix multiplication to solve a system of linear equations.

1. Enter the 3×3 identity matrix in matrix A.

2. Find the inverse matrix of the matrix B.

3. Solve the equation system.

$$\begin{cases} x + 2y + z = 8 \\ 2x + y - z = 1 \\ x + y - 2z = -3 \end{cases}$$

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{bmatrix}$$

Before Starting

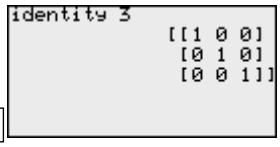
There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

As the Matrix feature is only available on the Advanced keyboard, this section does not apply to the Basic keyboard.

Step & Key Operation	Display	Notes

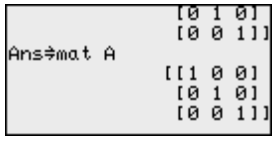
1-1 Set up 3×3 identity matrix at the home screen.

[▦] [2nd F] [MATRIX] [C] [0] [5] [3] [ENTER]



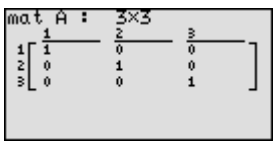
1-2 Save the identity matrix in matrix A.

[STO] [2nd F] [MATRIX] [A] [1] [ENTER]



1-3 Confirm that the identity matrix is stored in matrix A.

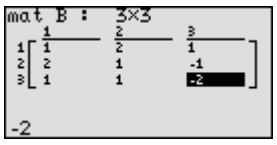
[2nd F] [MATRIX] [B] [1]



Step & Key Operation	Display	Notes

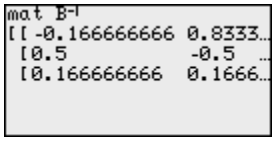
2-1 Enter a 3×3 matrix B.

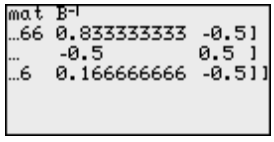
[2nd F] [MATRIX] [B] [2] [3] [ENTER] [3] [ENTER]
[1] [ENTER] [2] [ENTER] [1] [ENTER]
[2] [ENTER] [1] [ENTER] [(-)] [1] [ENTER]
[1] [ENTER] [1] [ENTER] [(-)] [2] [ENTER]



2-2 Exit the matrix editor and find the inverse of the square matrix B.

[2nd F] [QUIT] [CL]
[2nd F] [MATRIX] [A] [2] [2nd F] [X^{-1}] [ENTER]
[▶] (repeatedly)



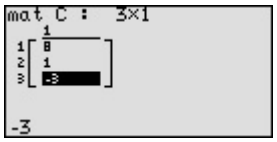


Some square matrices have no inverse and will generate error statements when calculating the inverse.

$$B^{-1}=\begin{bmatrix} -0.17 & 0.83 & -0.5 \\ 0.5 & -0.5 & 0.5 \\ 0.17 & 0.17 & -0.5 \end{bmatrix}$$

3-1 Enter the constants on the right side of the equal sign into matrix C (3×1).

[2nd F] [MATRIX] [B] [3] [3] [ENTER] [1] [ENTER]
[8] [ENTER] [1] [ENTER] [(-)] [3] [ENTER]



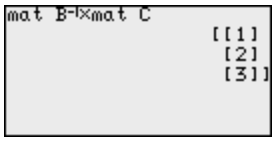
The system of equations can be expressed as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 8 \\ 1 \\ -3 \end{bmatrix}$$

Let each matrix B, X, C :
$BX = C$
$B^{-1}BX = B^{-1}C$ (multiply both sides by B^{-1})
$I = B^{-1}$ ($B^{-1}B = I$, identity matrix)
$X = B^{-1}C$

3-2 Calculate $B^{-1}C$.

[▦] [CL] [2nd F] [MATRIX] [A] [2]
[2nd F] [X^{-1}] [×] [2nd F] [MATRIX] [A] [3] [ENTER]



The 1 is the *x* coordinate, the 2 the *y* coordinate, and the 3 the *z* coordinate of the solution point.
$(x, y, z)=(1, 2, 3)$

3-3 Delete the input matrices for future use.

[2nd F] [OPTION] [C]
[2] [ENTER]
[2nd F] [QUIT]

The calculator can execute calculation of inverse matrix and matrix multiplication. A system of linear equations can be solved easily using the Matrix feature.