APPLYING

PRE-CALCULUS/CALCULUS

USING THE

SHARP

EL-9600

Graphing Calculator

DAVID P. LAWRENCE

Applying
PRE-CALCULUS/CALCULUS
using the
SHARP EL-9600
GRAPHING CALCULATOR

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This Teaching Resource has been developed specifically for use with the Sharp EL-9600 graphing calculator. The goal for preparing this book was to provide mathematics educators with quality teaching materials that utilize the unique features of the Sharp graphing calculator.

This book, along with the Sharp graphing calculator, offers you and your students 10 classroom-tested, topic-specific lessons that build skills. Each lesson includes *Introducing the Topic, Calculator Operations, Method of Teaching,* explanations for *Using Blackline Masters, For Discussion,* and *Additional Problems* to solve. Conveniently located in the back of the book are 33 reproducible *Blackline Masters*. You'll find them ideal for creating handouts, overhead transparencies, or to use as student activity worksheets for extra practice. *Solutions to the Activities* are also included.

We hope you enjoy using this resource book and the Sharp EL-9600 graphing calculator in your classroom.

Other books are also available:

Applying STATISTICS using the SHARP EL-9600 Graphing Calculator
Applying PRE-ALGEBRA and ALGEBRA using the SHARP EL-9600 Graphing Calculator
Applying TRIGONOMETRY using the SHARP EL-9600 Graphing Calculator
Graphing Calculators: Quick & Easy! The SHARP EL-9600

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Dedicated to my grandma, Carrie Lawrence

Special thanks to Ms. Marina Ramirez and Ms. Melanie Drozdowski
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Chapter one

EVALUATING LIMITS

Introducing the Topic

The concept of a limit is one of the basic building blocks of calculus. An understanding of limits is also necessary when investigating the behavior of a function near a vertical or horizontal asymptote and the end behavior of functions in precalculus.

The limits you and your students consider in this chapter fall into one of three categories:

- $\lim_{x \to \infty} f(x)$, the limit of a function $f(x)$ as x increases without bound. This limit is an indicator of the positive end behavior of the function.
- $\lim_{x \to -\infty} f(x)$, the limit of a function $f(x)$ as x decreases without bound. This limit is an indicator of the negative end behavior of the function.

When either of these two limits exist; that is, the values of $f(x)$ get closer and closer to a specific number L as x gets larger and larger or as x gets smaller and smaller, the line $y = L$, is a horizontal asymptote of the function.

- $\lim_{x \to a} f(x)$, the limit of a function $f(x)$ as x gets very close to, but does not equal, the value $x=a$. This limit describes the behavior of the function near x=a rather than at x=a. The limit of $f(x)$ as x approaches a exists and equals L, written $\lim_{x \to a} f(x) = l$, provided that for all values of x in the domain of $f(x)$, the values of $f(x)$ get closer and closer to L as x approaches a from each side of a.

When $\lim_{x \to a} f(x)$ does not exist in the sense that the values of $f(x)$ increase and/or decrease without bound as the values of x approach a, the line x=a is a vertical asymptote of the function.

This chapter investigates graphical and numerical methods of evaluating limits, provided those limits exist. These methods can, in many cases, give very accurate approximations of limits. However, they do not prove the existence of limits. You should consult a calculus text for methods of formal evaluation of limits.

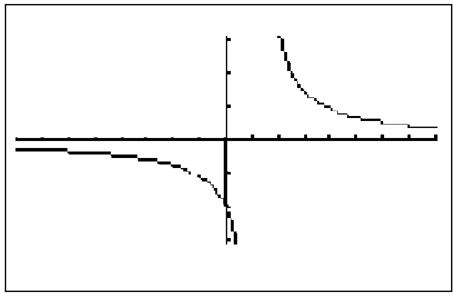
Calculator Operations

After turning your calculator on, prepare for the investigations in this chapter by setting the calculator to floating point decimal display by pressing [2ndF] [SET UP], touching **C FSE**, and double touching **1 Float Pt**. Set the calculator to rectangular coordinates by touching **E COORD** and double touching **1 Rect**. Press [2ndF] [QUIT] to exit the SET UP menu.

INVESTIGATING LIMITS GRAPHICALLY

Observing the graph of a function is useful for gathering information as to whether or not a limit exists. If the limit does exist, a graph is helpful in providing information that allows you to estimate the value of the limit or check an algebraically determined value.

Consider, for instance, the function $f(x) = (2x + 2)/(x^2 - 1)$. Press [Y=] [CL] to access and clear the Y1 prompt. Press [ENTER] [CL] to clear the remaining prompts. Construct a graph of $f(x)$ in the Decimal viewing window by first entering Y1= $(2x + 2)/(x^2 - 1)$ with the keystrokes [a/b] [2] [X/θ/T/n] [+] [2] [▶] [X/θ/T/n] [x^2] [–] [1], and then press [ZOOM], touch **A ZOOM**, touch ▼ on the screen, and touch **7 Dec** to see the graph.

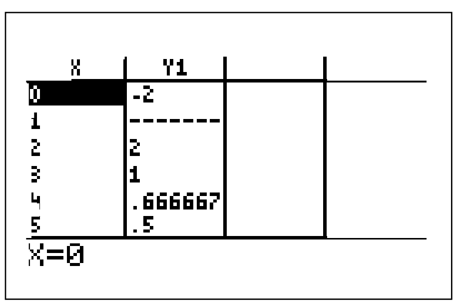


Notice that even though $(2x + 2)/(x^2 - 1)$ is not defined at $x = {}^-1$ (as evidenced by the hole in the graph at that point), the functional values appear to be getting closer and closer to $^-1$. A careful observation of the graph leads to the following estimates:

$\lim_{x \to {}^-\infty} f(x) \approx 0$, $\lim_{x \to {}^-1} f(x) \approx -1$, $\lim_{x \to 1} f(x)$ does not exist, and $\lim_{x \to \infty} f(x) \approx 0$.
It also appears that the line $y = 0$ is a horizontal asymptote and the line $x = 1$ is a vertical asymptote for this function.

INVESTIGATING LIMITS NUMERICALLY

Tables of functional values sometimes provide more detailed information than a graph when investigating limits. The Sharp EL-9600 has a TABLE feature to assist you in constructing a numerical table of values. Press $\boxed{\text{TABLE}}$ to access the TABLE feature.



Notice the table provides the x values and their corresponding y values according to Y1. You can change the table settings by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{TBLSET}}$.

You can change the table start value and the table step value.
Verify the following values using the the TABLE feature.

x gets smaller and smaller $\rightarrow$

x	$^-10$	$^-50$	$^-100$	$^-250$	$^-500$	$^-1000$	$^-10{,}000$
y	$^-.18182$	$^-.03922$	$^-.01980$	$.00797$	$^-.00399$	$^-.00200$	$^-.00020$

y = f(x) appears to get closer and closer to 0

This provides evidence that $\lim_{x \to {}^-\infty} f(x) = 0$.

x approaches $^-1$ *from the left* *x approaches* $^-1$ *from the right*

x	$^-1.05$	1.01	$^-1.001$	$^-1.0001$	$^-.9999$	$^-.999$	$^-.99$	$^-.90$
y	$^-.9756$	$^-.99502$	$^-.9995$	$^-.9999$	$^-1.0001$	$^-1.0005$	$^-1.0051$	$^-1.0526$

y=f(x) gets closer and closer to -1 from above *y=f(x)* gets closer and closer to $^-1$ from below

This provides evidence that $\lim_{x \to {}^-1} f(x) = {}^-1$.

Method of Teaching

Use *Blackline Master 1.1* to create an overhead or handout for investigating the limit of a function using a graph. Be certain students understand that this method provides estimates of limits and does not constitute proof that a limit exists or does not exist. Use *Blackline Master 1.2* to create an overhead or handout for numerically investigating the limit of a function using the TABLE feature. If students cannot establish a pattern using the values indicated on the *Blackline Masters,* they should evaluate the function at other values of *x* until a pattern is established or until they can determine that the function either increases or decreases without bound. Use *Blackline Master 1.3* to create a worksheet for the students. Use the topics *For Discussion* to supplement the worksheets.

Using Blackline Master 1.3

Problem 1

At points where there is a *jump* discontinuity in the function (the limit from the left differs from the limit from the right), you may find it necessary to set the calculator to dot mode by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{FORMAT}}$, touching **E STYLE1**, and double touching **2 Dot**. Do this whenever the pieces of the graph appear connected at the point of discontinuity.

Problem 2

Discuss with students why the line $x = 3$ is a vertical asymptote and why the line $y = 1$ is a horizontal asymptote for $f(x)$. Also discuss the difference in the nature of the two discontinuities at $x = 3$ and $x = {}^{-}3$.

Problem 3

The equation in this example is called a logistics equation, and it represents limited population growth. Discuss with students why the independent variable in the function must be x for graphing. After drawing the graph, students can press TRACE, and press ◀ or ▶ to trace the graph and estimate $\lim_{x \to \infty} p(x)$ to be 10,000. The answer to the first question is obtained by using the TABLE feature or the CALC feature when viewing the graph. Students should let x take on larger and larger values to obtain $\lim_{x \to \infty} p(x)=10{,}000$.

For Discussion

A function $f(x)$ is said to be *continuous* at a value $x = a$ if:

- $f(a)$ exists,
- $\lim_{x \to a} f(x)$ exists, and
- $\lim_{x \to a} f(x) = f(a)$

Discuss with students why the function in **Blackline Master 1.3**, $f(x)=|x + 1|/(x+1)$, is continuous at all values of x except $x={}^{-}1$. Also discuss why the function $f(x)=(x^2+2x - 3)/(x^2 - 9)$, is continuous at all values except $x = {}^{-}3$ and $x = 3$, and why the function $p(t)=10{,}000/(1 + 15e^{-t/4})$, is continuous at all values of t.

Additional Problems

In problems 1-4, evaluate the limits using numerical methods. Use a graph of the function to estimate the values of x where $f(x)$ is not continuous.

1. $\lim_{x \to -2} f(x)$ if $f(x) = (8 + x^3)/(x + 2)$

2. $\lim_{x \to 2.5} f(x)$ if $f(x) = (x - 2.5)/|2.5 - x|$

3. $\lim_{x \to 0} f(x)$ and $\lim_{x \to \infty} f(x)$ if $f(x) = (1 + x)^{1/x}$

4. $\lim_{x \to \infty} f(x)$ if $f(x)=(2300x)/(500 - x)$

5. Mary is taking a typing class. Suppose the number of words per minute W that Mary can type after t weeks of practice is given by the equation $W(t)=85(1 - e^{-0.3t})$.
   a. If the class lasts 6 weeks, how many words per minute can Mary type at the end of the class?
   b. If Mary gets a job that requires her to type (and therefore continue practicing), will she ever be able to type 100 words per minute?

Chapter two

DERIVATIVES

Introducing the Topic

The derivative is one of the fundamental tools of calculus used to study functions and solve problems. The derivative of a function tells us the rate at which the values of $f(x)$ are changing as x changes. The ratio $[f(x) - f(x)]/(x - a)$ gives the *average rate of change* of a function $f(x)$ with respect to the variable x. Provided it exists, $\lim_{x \to a} [f(x) - f(a)]/(x - a)$ is called the *instantaneous* rate of change of $f(x)$ with respect to the variable x at a, or more simply, the *derivative* $f'(a)$. If the limit does not exist, we say that $f(x)$ is not differentiable at a.

Graphically, the average rate of change is the slope of the secant line joining the points $(x, f(x))$ and $(a, f(a))$ for $x \neq /a$, while the derivative $f'(a)$ gives the slope of the tangent line to the function $f(x)$ at $x = a$. Chapter 3 offers a program to further investigate the geometrical interpretation of a derivative as the slope of a the tangent line.

Calculator Operations

Prepare for the investigations in this chapter by setting the calculator to radian measure by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touching **B DRG**, and double touching

2 Rad. Set the calculator to floating point decimal display by touching **C FSE** and double touching **1 Float Pt**. Set the calculator to rectangular coordinates by touching **E COORD** and double touching **1 Rect**. Press [2ndF] [QUIT] to exit the SET UP menu.

DERIVATIVES USING THE LIMIT DEFINITION

Consider the function $f(x) = x^2$. To find the derivative of this function at $x = 2$ using the definition $f'(2) = \lim_{x\to 2} [f(x) - f(2)]/(x - 2) = \lim_{x\to 2} (x^2 - 4)/(x - 2)$, provided the limit exists, use the TABLE feature that was discussed in Chapter 1 to verify the entries for the values of the quotient $(x^2 - 4)/(x - 2)$ in the table below: (Table entries are rounded to five decimal places.)

x approaches 2 from the left→ ←*x* approaches 2 from the right

x	1.9	1.99	1.999	1.9999	2.0001	2.001	2.01	2.1
y	3.9	3.99	3.999	3.9999	4.0001	4.001	4.01	4.1

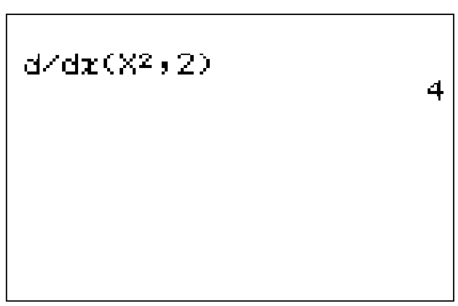
The quotient gets closer and closer to 4 from below

The quotient gets closer and closer to 4 from above

It certainly appears that $\lim_{x\to 2} (x^2 - 4)/(x - 2) = f'(2) = 4$

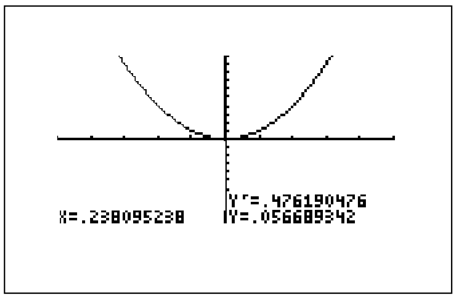
DERIVATIVES USING THE d/dx FUNCTION

The calculator has a built-in function denoted by *d*/*dx* that uses numerical methods to estimate the derivative of a function at a given value. The entry form of the derivative function is $d/dx\,(f(x), a)$. For instance, to estimate $f'(2)$ for $f(x) = x^2$ using the derivative function, press [+ − × ÷] [MATH], touch **A CALC**, double touch **05 *d*/*dx*(** press [X/θ/T/*n*] [x^2] [,] [2] [)] to input: $d/dx(x^2, 2)$. Press [ENTER] to compute.



DERIVATIVES USING THE DERIVATIVE TRACE

Your calculator can also display values calculated by the derivative function on the graphics screen as you trace the function. The values that are displayed are the values calculated by the derivative function *d*/*dx*. To activate this *y*' trace, press $\boxed{\text{2ndF}}$ $\boxed{\text{FORMAT}}$, touch **D Y'**, and double touch **1 ON**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the FORMAT menu. Now, press $\boxed{\text{Y=}}$ $\boxed{\text{CL}}$ and enter the function for Y1 by pressing $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{x^2}$. Draw the graph by pressing $\boxed{\text{WINDOW}}$ $\boxed{\text{EZ}}$, touching **5**, touching ▼ on the screen, double touching **7 -5<x<5**, and double touching **4 -10<Y<10**. Press $\boxed{\text{TRACE}}$ to activate the derivative trace and press $\boxed{▶}$ to observe the values calculated by the derivative trace as the trace cursor moves along the graph.



Turn off the derivative trace by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{FORMAT}}$, touch **D Y'**, and double touch **2 OFF**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the FORMAT menu.

Method of Teaching

Use *Blackline Master 2.1* to create an overhead or handout for investigating the derivative of a function using the limit definition. Use *Blackline Master 2.2* to create an overhead or handout for investigating the built-in derivative function *d*/*dx*, and use *Blackline Master 2.3* to create an overhead or handout for using the derivative trace.

Be certain that students understand the *d*/*dx function* and the *y' trace* give approximations to the derivative of the function at *y*, <u>not</u> the exact value of $\lim_{x \to a} [f(x) - f(a)]/(x - a)$. Use the topics *For Discussion* to supplement the worksheets.

Using Blackline Master 2.1

The first problem demonstrated above under *Calculator Operations* is addressed on *Blackline Master 2.1* as Activity 1. In Activity 2, students should be able to determine from the graph of $(2x^3 - 2)/(x - 1)$ that the limit as x approaches 1 exists. Be certain they notice that this quotient is not defined at $x = 1$, as evidenced by the hole in the graph at $x = 1$.

Activity 3 asks the students to find $f'({}^{-}3)$. Discuss with them that the limit definition of the derivative gives $f'(3) = \lim_{x\to{}^{-}3} [f(x) - f({}^{-}3)]/(x - 3) = \lim_{x\to{}^{-}3} (2x^3 + 54)/(x + 3)$ provided the limit exists. After determining from the graph that the limit certainly seems to exist, students should create a table of values, letting x approach ${}^{-}3$ from both the left and right, to see that $f'({}^{-}3)$ appears to be 54.

Using Blackline Master 2.2

The second problem demonstrated above under *Calculator Operations* is addressed on *Blackline Master 2.2* as Activity 1. In Activity 2, students should approximate the value of $f'({}^{-}1)$ with $d/dx(2x^3, 1)$. Activity 3 asks students to determine that $f'(0)$ does not exist for $f(x)=|x|$, have them construct a graph of $y=|x|$, and point out that the derivative of a function does not exist where the graph comes to a "*v*". This is a good example to show that continuity does not imply differentiability since $|x|$ is continuous, but not differentiable, at $x = 0$.

Using Blackline Master 2.3

The third problem demonstrated above under *Calculator Operations* is addressed on *Blackline Master 2.3* as Activity 1. In Activity 2, students should look for a pattern in the $y' = f'(x)$ values as they relate to the x values or the $y = f(x)$ values to discover a rule for $f'(x)$. Discuss with students that this method only provides an estimate for the rule for $f'(x)$, not a proof of that rule or that $f'(x)$ exists for all values of x. In Activity 3, students should realize that $f'(0)$ does not exist because the values of $d/dx(x1/3/(x^2+1), 0)$, as displayed by the f' trace, increase without bound as x approaches 0 from either side of 0.

For Discussion

The derivative of a function $f(x)$ is often defined as:

$$f'(x) = \lim_{\Delta x \to 0} [f(x + \Delta x) - f(x)]/\Delta x.$$

If you substitute $x = a + \Delta x$ in the definition of the derivative given at the beginning of this chapter, $f'(a) = \lim_{x \to a} [f(x) - f(a)]/(x - a)$, you obtain $f'(a) = \lim_{\Delta x \to 0} [f(a + \Delta x) - f(a)]/\Delta x$, the derivative of $f(x)$ evaluated at $x = a$. Discuss with the students that Δx used in this definition has the same meaning as the *Dx* used in the *d/dx function* and the *y' trace*.

Additional Problems

1. Use the limit definition of the derivative to determine if $f'(1)$ exists for $f(x) = 2x - 3$. If $f'(1)$ exists, find its value. If it does not exist, explain why not.

2. Use the limit definition of the derivative to determine if $f'(1)$ exists for $f(x) = |x - 1|$. If $f'(1)$ exists, find its value. If it does not exist, explain why not.

3. Use values obtained with the derivative function *d/dx* to determine if $f'(0)$ exists for $f(x) = 2x/(x + 1)$. If $f'(0)$ exists, estimate its value. If it does not exist, explain why not.

4. Use values obtained with the derivative function *d/dx* to determine if $f'(0)$ exists for $f(x) = \sin(\pi x)$. If $f'(0)$ exists, estimate its value. If it does not exist, explain why not.

5. For each of the following, enter the X parameters in the WINDOW screen and then press $\boxed{\text{ZOOM}}$, and double touch **1 Auto** to construct a graph of the function $y = f(x)$ over the indicated interval, and then use the *y' trace* to complete the table of values for $y' = f'(x)$. (Round table entries to 3 decimal places.) Use the pattern you view in the table of values to determine a formula for $f'(x)$.

a. $f(x)=e^{-x}$
($^-6.3 < x < 6.3$)

x	y	y'
$^-4$		
$^-3$		
$^-2$		
$^-1$		
0		
1		
2		
3		
4		

b. $f(x)= 1/2\ x^2$
($^-6.3 < x < 6.3$)

x	y	y'
$^-4$		
$^-3$		
$^-2$		
$^-1$		
0		
1		
2		
3		
4		

c. $f(x)=\ln x$, $x>0$
($0 < x < 12.6$)

x	y	y'
.5		
1		
2		
3		
4		
5		
6		
7		
8		

Chapter three

TANGENT LINES

Introducing the Topic

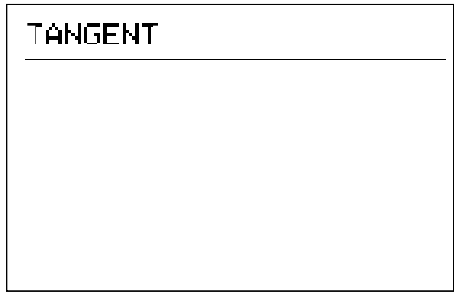
In this chapter, you and your students will learn how to program the Sharp EL-9600 graphing calculator, execute the program and use the program to find tangent lines to a curve. Further, you and your students will learn how to draw a tangent line to a curve at a point. A tangent line to a curve is the line that intersects the graph in only one point and its slope represents the slope of the curve at that point. The TANGENT program computes the tangent line to a curve at a particular point. The function is entered for Y1.

Calculator Operations

Turn the calculator on and press 2ndF PRGM to enter the programming menu. The menu consists of commands to execute, edit, and create new programs. Touch **C NEW** and press ENTER to open a new program.

```
TITLE?
```

The calculator is now locked in ALPHA mode and is prepared to accept a name for the new program. Name the new program TANGENT by pressing T A N G E N T ENTER .



You can now enter in the TANGENT program. Please note that you must press ENTER at the end of each line. If you make a mistake, use the calculator's editing features to correct the error. Enter the following program:

PROGRAM KEYSTROKES

(Input the point at which the tangent line is to be drawn)

Input X 2ndF PRGM A 3 X/θ/T/*n* ENTER

(Find the slope of the tangent line)

d/dx(Y1, X)⇒M MATH A 0 5 VARS A ENTER A 1 , X/θ/T/*n*) STO ALPHA M ENTER

(Compute the point of intersection for the curve and tangent line)

Y1(X)⇒Y VARS A ENTER 1 (X/θ/T/*n*) STO ALPHA Y ENTER

(Compute the *y* intercept)

$^{-}$M•X+Y⇒B (–) ALPHA M × X/θ/T/*n* + ALPHA Y STO ALPHA B ENTER

(Display the equation for the line)

ClrT 2ndF PRGM C 1 ENTER

Print "TANGENT LINE=	[2ndF] [PRGM] [A] [1] [2ndF] [PRGM] [2] [2ndF] [ALPHA] [T] [A] [N] [G] [E] [N] [T] [SPACE] [L] [I] [N] [E] [=] [ENTER]
Print "Y=MX+B	[2ndF] [PRGM] [1] [2ndF] [PRGM] [2] [ALPHA] [Y] [ALPHA] [=] [ALPHA] [M] [X/θ/T/*n*] [+] [ALPHA] [B] [ENTER]
Print "M=	[2ndF] [PRGM] [1] [2ndF] [PRGM] [2] [ALPHA] [M] [ALPHA] [=] [ENTER]
Print M	[2ndF] [PRGM] [1] [ALPHA] [M] [ENTER]
Print "B=	[2ndF] [PRGM] [1] [2ndF] [PRGM] [2] [ALPHA] [B] [ALPHA] [=] [ENTER]
Print B	[2ndF] [PRGM] [1] [ALPHA] [B] [ENTER]
End	[2ndF] [PRGM] [6] [ENTER]

```
TANGENT
Input X
d/dx(Y1,X)⇒M
Y1(X)⇒Y
-M*X+Y⇒B
ClrT
Print "TANGENT LINE=
```

```
Print "TANGENT LINE=
Print "Y=MX+B
Print "M=
Print M
Print "B=
Print B
End
```

Press [2ndF] [QUIT] to save the program and exit the editing mode.

Before executing the TANGENT program, you need to enter the function of interest for Y1. Enter $f(x) = x^3 - x^2 + 1$ for Y1 by pressing [Y=] [CL] [X/θ/T/*n*] [a^b] [3] [▶] [–] [X/θ/T/*n*] [x^2] [+] [1] [ENTER].

```
Y1=X^3 -X²+1
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
```

Execute the TANGENT program by pressing 2ndF PRGM , touching **A EXEC**, and double touching **TANGENT**. Enter an X value for the point at which you desire the tangent line to be found. Enter an X of 1 by pressing 1 ENTER . You should then see the following equation for the tangent line to the curve at $x = 1$.

```
Y=MX+B
M=
                    1
B=
                    0
```

You can repeat this process for other x values. Press ENTER to execute the program over and over again. Press CL to clear the screen.

If you receive an error statement, press ◀ or ▶ to go to the line within the program in which the error occurs. Compare your line with the correct one above to find the error. Correct the error using the editing features of the calculator and save the program by pressing 2ndF QUIT . Try executing the program again.

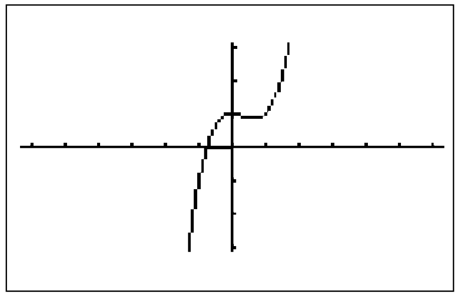
Method of Teaching

Use *Blackline Masters 3.1* and *3.2* to create overheads for entering and executing the TANGENT program. Go over in detail how to enter the program and what the different program lines are doing. Have the students enter the program and execute it (correcting any errors). Use *Blackline Masters 3.3* to create a worksheet for the students on how to draw tangent lines on a graph. Use the topics *For Discussion* to supplement the worksheets.

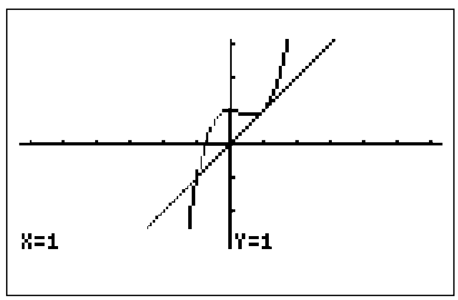
Using Blackline Master 3.3

To draw the tangent line on a displayed graph, you must first enter the graph for Y1. Enter the function $f(x) = x^3 - x^2 + 1$ for Y1 by pressing Y= CL X/θ/T/n a^b 3 ▶ – X/θ/T/n x^2 + 1 ENTER .

Graph the function by pressing $\boxed{\text{ZOOM}}$, touching **A ZOOM**, touching ▼ on the screen, and touching **7 Dec**.



Draw the tangent line at x = 1 by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{DRAW}}$, touching **A DRAW**, double touching **5 T_line(**, move the tracer right to x =1 by pressing $\boxed{\blacktriangleright}$ repeatedly, and then press $\boxed{\text{ENTER}}$.



For Discussion

You and your students can discuss what occurs when the tangent line is horizontal.

Additional Problems

Find and graph the tangent lines for the following functions at the given point. Remember to graph in the decimal window so your tracer will find the *x* integer.

1. $f(x) = x^2$ at $x = 2$

2. $f(x) = \sin x$ at $x = 1$

3. $f(x) = \tan x$ at $x = {}^{-}.5$

4. $f(x) = \sqrt{x}$ at $x = 1$

5. $f(x) = -(x - 2)^2 + 1$ at $x = 2$

Chapter four

GRAPHS OF DERIVATIVES

Introducing the Topic

In this chapter, you and your students will explore connections between the graph of a function and the graph of the first and second derivatives of the function.

Calculator Operations

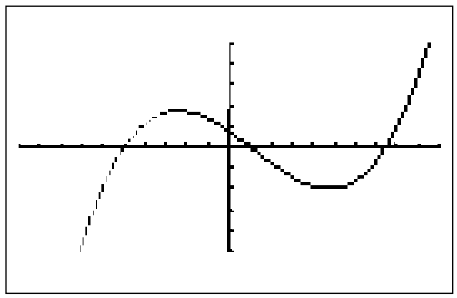
Prepare for the investigations in this chapter by setting the graphing calculator to radian measure by pressing 2ndF SET UP , touching **B DRG**, and double touching **2 Rad**. Set the calculator to floating point decimal display by touching **C FSE** and double touch **1 FloatPt**. Set the calculator to rectangular graphing by touching **E COORD**, and double touching **1 Rect**. Press 2ndF QUIT to exit the SET UP menu.

Also, choose sequential graphing by pressing 2ndF FORMAT , touching **F STYLE2**, and double touching **1 Sequen**. Set the calculator to connected mode by touching **E STYLE1** and double touching **1 Connect**. Press 2ndF QUIT to exit the FORMAT menu.

The built-in derivative function d/dx can be used to draw the graph of the derivative of a function at all points where the derivative exists.

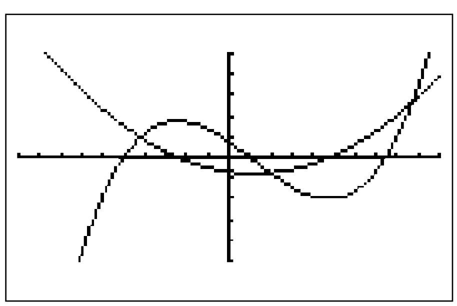
Using the calculator, students can easily draw the graphs of many functions and their derivatives to discover connections between the graphs of $f(x)$, $f'(x)$, and $f''(x)$. For instance, suppose that $f(x)=2x^3 - 7x^2 - 70x + 75$. Press [Y=] [CL] to access and clear Y1. Press [ENTER] [CL] to clear additional Y prompts. Enter $f(x)$ for Y1 by pressing [2] [X/θ/T/*n*] [a^b] [3] [▶] [–] [7] [X/θ/T/*n*] [x^2] [–] [7] [0] [X/θ/T/*n*] [+] [7] [5] .

Graph the function by pressing [WINDOW] [EZ], touching **5**, double touching **5 -10<*X*<10**, and double touching **1 -500<X<500**.



To determine the point at which the relative maximum occurs, press [TRACE] [2ndF] [CALC], and double touch **4 Maximum**. The maximum occurs at X= ⁻2.4427, Y = 175.07. Find the point at which the relative minimum occurs with [2ndF] [CALC], and double touch **3 Minimum**. The minimum occurs at X= 4.776, Y = ⁻201.1084. Combining this information with a view of the graph, we see that $f(x)$ is increasing from ⁻∞ to ⁻2.4427 and from 4.776 to ∞ while $f(x)$ is decreasing for x between ⁻2.4427 and 4.776.

Next, construct the graph of $f'(x)$. Press [Y=] [ENTER] and input d/dx(Y1) in the Y2 location with the keystrokes [MATH], touch **A CALC**, double touch **05 *d/dx* (** press [VARS] [ENTER], touch **A XY**, double touch **1 Y1**, and press [)] [ENTER]. Press [GRAPH] to obtain the graphs of $f(x)$ and $f'(x)$.



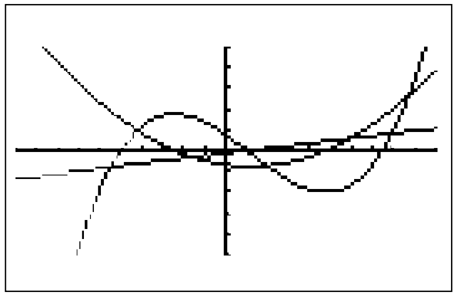
We now want to find the two x-intercepts of $f(x)$. Press TRACE ▼ to place the tracer on the graph of the derivative. Then, press 2ndF CALC and double touch **5 X_Incpt** to obtain X = 4.77606. Press 2ndF CALC and double touch **5 X_Incpt** again to obtain the other x-intercept at X = $^-2.44273$. Comparing these values to the x-coordinates of the points at which the maxima and minima of $f(x)$ occur, we see they are almost identical. Calculus theory tells us that these are exactly the same values. However, you may view a slight difference in trailing decimal places due to the numerical approximation routines used by the calculator. For convenience, we will round answers to three decimal places.

Where is $f'(x)$ positive? The graph of the derivative is above the x-axis for $x<^-2.443$ and $x>4.776$. Notice this is where the graph of the function $f(x)$ is increasing. Where is $f'(x)$ negative? The graph of the derivative is below the x-axis for $^-2.443<x<4.776$. Notice this is where the graph of $f(x)$ is decreasing.

Next, find the minimum point of $f'(x)$ by first making sure the trace cursor is on the graph of the derivative, pressing 2ndF CALC, and double touching **3 Minimum**. The minimum of the derivative occurs at the point X = 1.167, Y = $^-78.167$. Look at the graph of the function of the derivative and observe that this appears to be the point at which the function "bends a different way"; that is, the point at which $f(x)$ changes concavity and is called the point of inflection.

Find the point of inflection directly by moving the cursor to the original function and pressing 2ndF CALC, touching ▼ on the screen, and touching **7 Inflec**.

Let's now add the graph of the second derivative, $f''(x)$, to the picture. Press Y= ENTER ENTER, and input d/dx(Y2) in the Y3 location with the keystrokes MATH, touch **A CALC**, double touch **05 *d/dx*(** press VARS ENTER, touch **A XY**, double touch **2 Y2**, and press) ENTER. Press GRAPH to obtain the graphs of $f(x)$, $f'(x)$, and $f''(x)$.



(Notice that the graph of *f'*(*x*) takes longer to draw than the graph of *f*(*x*) and that the graph of *f"*(*x*) takes even longer to appear on the screen. This is because the calculator is determining functional values of the derivatives using numerical approximations before plotting the points and connecting them to draw the graph. In fact, the graph of *f"*(*x*) may at times appear "jagged" for this reason. Students should realize that since this function is a cubic, its derivative is a quadratic, and the second derivative is therefore a line. Your students may sometimes prefer entering algebraically-calculated derivatives rather than using the calculator-generated derivatives.)

Where is *f"*(*x*) zero? After pressing TRACE ▼ ▼ to place the tracer on the graph of the second derivative, press 2ndF CALC and double touch **5 X_Incpt** to find that *f"*(*x*)= 0 at X= 1.167. Calculus theory tells us that this is exactly the *x*-value of the point where the function *f*(*x*) changes concavity; that is, the inflection point.

Find the *y*-value of this point by pressing ▲ ▲ to move the cursor to the original function. Press 2ndF CALC, double touch **1 Value**, and enter 1.167 by pressing 1 • 1 6 7 ENTER. The calculator provides a *y* value of ⁻13.045. To the left of the point (1.167, ⁻13.045) the function is concave down (curved downward) and to the right of the inflection point, *f*(*x*) is concave up (curved upward).

The connections we have discovered between the graphs of *f*(*x*), *f'*(*x*), and *f"*(*x*) are summarized in the tables on the next page.

Interval	$f(x)$ is	$f'(x)$ is	$f''(x)$ is
$x < -2.443$	increasing, concave down	positive	negative
$^-2.443 < x < 1.167$	decreasing, concave down	negative	negative
$1.167 < x < 4.776$	decreasing, concave up	negative	positive
$x > 4.776$	increasing, concave up	positive	positive

x-value of point	$f(x)$ has	$f'(x)$ has	$f''(x)$ has
$x = {^-2.443}$	relative maximum	x-intercept	—
$x = 1.167$	inflection point	minimum	x-intercept
$x = 4.776$	relative minimum	x-intercept	—

Method of Teaching

Use *Blackline Masters 4.1, 4.2,* and *4.3* to create overheads or handouts for investigating how the first and second derivatives of a function can be used to find where the graph of the function is increasing/decreasing and changes concavity. Also, use the Blackline Masters for investigating connections between the graph of a function and its first and second derivatives. Use *Blackline Masters 4.4* to create a worksheet for the students. Inform the students of inaccuracies in the calculator-generated values of higher derivatives using the *d/dx*(operation, and they need to find the derivatives by hand to enter them in Y2, etc.

Using Blackline Master 4.4

In Activity 1, students should trace the graph of Y3 to see that it lies on the x-axis except for at x=0 where $f''(x)$ does not exist. When students use the *d/dx* function to construct the graph of the second derivative of $\sqrt{(9-X^2)}$ in Activity 2, inaccuracies occur. In the two figures below, the one on the left shows the graph of Y1= $\sqrt{(9-X^2)}$, Y2=*d/dx*(Y1), and Y3=*d/dx*(Y2). The figure on the right shows the graphs of Y1= $\sqrt{(9-X^2)}$, Y2=-X/ $\sqrt{(9-X^2)}$= $f'(x)$ obtained by the power and chain rules, and Y3=*d/dx*(Y2). Both figures were drawn using the default viewing window.

For Discussion

The derivative function *d/dx* is very reliable when graphing the first derivative of a function. However, if you want it to graph second and/or higher order derivatives that are computed from a calculator-generated derivative, inaccuracies may sometimes result due to limitations of the numerical approximation techniques and the technology.

Additional Problems

Construct the graphs of $f(x)$, $f'(x)$, and $f''(x)$ for each of the following functions. Use these graphs of the first and/or second derivatives to identify where the function $f(x)$ is increasing, decreasing, concave up, concave down, and where any relative maxima or minima occur.

1. $f(x) = 5x^2 - x^3 + 4x - 2$

2. $f(x) = e^{-x^2}$

3. $f(x) = \sqrt{x}$

4. $f(x) = |x^3|$

Chapter five

OPTIMIZATION

Introducing the Topic

In this chapter, you and your students will learn to apply procedures for finding maxima and minima to solve “real-world” problems. The calculator’s CALC function can be used to approximate such values with a high degree of accuracy in a precalculus course, and finding exact values of maxima and minima is one of the most important applications of first derivatives in a calculus course.

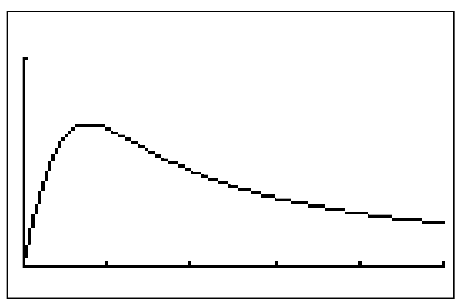
Calculator Operations

Prepare for the investigations in this chapter by setting the calculator to radian measure by pressing [2ndF] [SET UP], touching **B DRG**, and double touching **2 Rad**. Set the calculator to floating point decimal display by touching **C FSE** and double touch **1 FloatPt**. Set the calculator to rectangular graphing by touching **E COORD**, and double touching **1 Rect**. Press [2ndF] [QUIT] to exit the SET UP menu.

Instructions given in this chapter are appropriate for either a precalculus or calculus course. However, if you are using this manual in a calculus course, you can have students enter the function in each problem in Y1, *d*/*dx*(Y1) in Y2, and graph the function and the calculator-generated derivative in an appropriate viewing window. Students should then enter their algebraically-determined derivative in Y3 and press GRAPH . If only two graphs are observed, it is very probable that the algebraically-determined derivative has been correctly computed.

Consider the following application. A new product was introduced through a television advertisement appearing during the Super Bowl. Suppose that the proportion of people that purchased the product *x* days after the advertisement appeared is given by $f(x) = (5.3x)/(x^2 + 15)$. When did maximum sales occur and what proportion of people purchased the product at that time?

To answer this question using a graph of $f(x)$, first find a suitable domain and range for the problem situation. Since *x* is the number of days after the advertisement appeared, $x \geq 0$, and because *y* is a proportion, $0 \leq y \leq 1$. Next, press Y= CL to clear the Y1 prompt. Press ENTER CL to clear additional prompts. Enter $f(x)$ in the Y1 location with the keystrokes a/b 5 • 3 X/θ/T/*n* ▶ X/θ/T/*n* x^2 + 1 5 . Let's examine the graph for the first 25 days after the advertisement appeared. Press WINDOW , enter Xmin= 0, Xmax= 25, Xscl= 5, Ymin= 0, Ymax= 1, Yscl= 1. Press GRAPH to obtain:

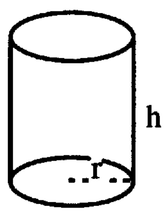


When did maximum sales occur and what proportion of people purchased the product at that time? Press 2ndF CALC and double touch **4 Maximum** to find X = 3.873, Y= 0.684. We see from the graph that a relative (local) maximum occurs at this point. Is it the absolute maximum? Press WINDOW and change Xmax to 100. Press GRAPH to view the graph of the function. Repeat the procedure for Xmax = 500. The graph certainly does not appear to have any functional values greater than Y= 0.684.

(Calculus students should realize that since $f(x)$ is a continuous function, the absolute maximum occurs at a point where either $f'(x)= 0$ or $f'(x)$ does not exist or at an endpoint of the interval. They can therefore obtain the definite answer that the absolute maximum occurs at X= 3.873, Y= 0.684.) You may wish to have students express their answers to problems in this chapter in sentence form. If so, an appropriate answer is "The maximum sales occurred 3.873 days after the advertisement first appeared, and the proportion of people that purchased the product at that time is .684."

Let's look at another example. A metal container with no top in the form of a right circular cylinder is being designed to hold 185 in^3 of liquid. If the material for the container costs 14¢ per square inch and the cost of welding the seams around the circular bottom and up the side cost 5¢ per inch, find the radius of the container with the smallest cost. What is the minimum cost?

You will need these formulas:



Cylinder:	Circular Base:
Volume= $\pi r^2 h$	Area= πr^2
Lateral Surface Area= $2\pi rh$	Circumference= $2\pi r$

Now, the area of the container = area of lateral surface + area of bottom = $2\,\pi rh + \pi r^2$

length of welds = $h + 2\pi R$

\$ cost of container = .14(area of container) + .05(length of welds)

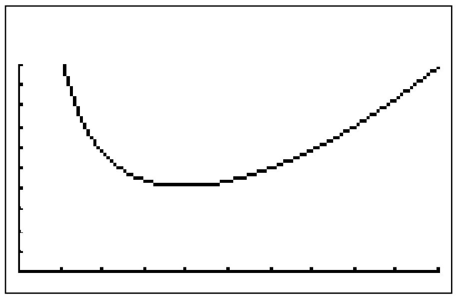
$= .14(2\pi rh + \pi r^2) + .05(h + 2\pi r)$

Next, we know that $183 = \pi r^2 h$, so $h = 183/(\pi r^2)$.

Substituting this in the cost equation, we find \$ cost of container $= .14(2\pi r(183/(\pi r^2)) + \pi r^2) + .05(183/(\pi r^2) + 2\pi r)$.

Recall that the independent variable in the graphing mode must be called x. Press $\boxed{\text{Y=}}$ and clear any previously-entered functions with $\boxed{\text{CL}}$, and either type in the \$ cost of container in its current form with $r = x$ or type in the simplified

version, $51.24/x + .14\pi x^2 + 9.15/(\pi x^2) + .1\pi x$, in Y1 with the keystrokes
[a/b] [5] [1] [•] [2] [4] [▶] [X/θ/T/*n*] [▶] [+] [•] [1] [4] [2ndF] [π] [X/θ/T/*n*] [x^2] [+] [a/b] [9] [•] [1] [5] [▶] [2ndF] [π] [X/θ/T/*n*] [x^2] [▶] [+] [•] [1] [2ndF] [π] [X/θ/T/*n*] . What window settings do we use? Since x is the radius of the container, we know that $x > 0$. The cost Y1 will also be greater than 0. A little experimenting leads to a viewing window of $0 < x < 10$ and $0 < y < 50$. Enter the viewing window and press [GRAPH] to view the graph of the cost function:



To find the minimum cost, press [2ndF] [CALC] and double touch **3 Minimum** to obtain X= 3.799, Y= 21.231. Changing Xmax to increasingly larger values shows that the costs continue to increase past this point. Therefore, the radius of the container with the smallest cost is 3.799 inches. The minimum cost of the container is $21.23.

Method of Teaching

Use *Blackline Master 5.1* to create an overhead or handout for investigating a maximization problem. Use *Blackline Master 5.2* to create an overhead or handout for investigating a minimization problem. Use *Blackline Master 5.3* to create an overhead or handout for further investigation of optimization problems. Caution students that they should always check their answers to “real-world” problems to see if they make sense. Use the topics *For Discussion* to supplement the worksheets.

Using Blackline Master 5.3

The problems discussed above under *Calculator Operations* are shown on *Blackline Master 5.1 and 5.2*. *Blackline Master 5.3* provides two additional activities. In Activity 2, students need to realize that part of the region that is along the wall of the house requires no fence. They then need to minimize the perimeter function $f(w) = 2w + 675/w$. Students should express this function in terms of x, draw the graph in a viewing window such as $0 < x < 75$, $0 < y < 200$, and press $\boxed{\text{2ndF}}$ $\boxed{\text{CALC}}$ and double touch **3 Minimum** to find the minimum width.

Some students may choose to work with the function $f(l\,) = 2(675/l\,) + \text{l}$. The method is the same.

For Discussion

Discuss the terms relative (local) *maxima* or *minima* and absolute *maximum* or *minimum* with students. Calculator-generated graphs only show a portion of the graph of a function, so we can just verify that relative maxima or minima exist. While calculus students have methods to justify that the y-coordinate of a point is an absolute maximum or minimum, precalculus students can only form an "educated" opinion by observing the behavior of the graph for increasing values of x.

Additional Problems

1. The sum of two whole numbers is 52. What is the smallest possible value of the sum of their squares?

2. Rancher Johnson has 250 meters of fencing. What is the largest possible area of a rectangular corral that he can enclose with the fencing? Allow 3 meters for a gate (on one of the longer sides of the corral) that is not made from the fencing.

3. The formula $h = {}^{-}16t^2 + v_o t + h_o$ gives the height h feet above the ground that an object propelled vertically upward from an initial height h_o feet with an initial velocity of v_o feet per second is t seconds after it is propelled upward. (Assume air resistance is negligible.)

   a. Find the maximum height attained by a toy rocket that is shot vertically upward from ground level at an initial velocity of 30 feet per second.

   b. Find the time it takes a ball that is thrown vertically upward with an initial velocity of 12 feet per second from a cliff that is 200 feet above ground level to reach its maximum height.

4. A voltage, measured in volts, applied to a certain electronic circuit for t seconds is given by the equation $v(t) = 1.5\, e^{\cos(2t)}$. What is the maximum voltage during a 3 second time interval?

5. A rectangular bin, designed to hold 16 cubic feet of grain, is to be constructed with a square base and no top. The cost for the base of the bin is 20¢ per square foot and the cost for each of the sides is 12¢ per square foot. Find the dimensions that minimize construction costs.

Chapter sIX

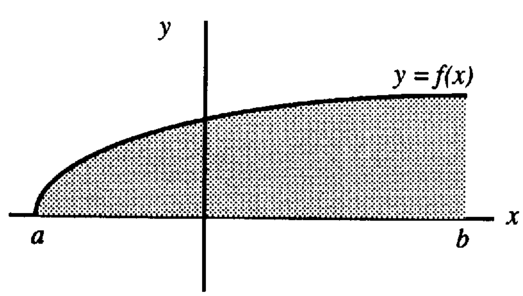
SHADING AND CALCULATING AREAS REPRESENTED BY AN INTEGRAL

Introducing the Topic

In this chapter, you and your students will learn to use the calculator's numerical integrate function to approximate the definite integral of a function over a specified interval. Calculus students learn that the connection between definite integrals and the geometric concept of area is this:

If f is a continuous function for $a \le x \le b$ and $f(x) \ge 0$ for $a \le x \le b$, the area of the region between $y = f(x)$ and the x-axis from $x = a$ to $x = b$ is given by $\int_a^b f(x)dx$.

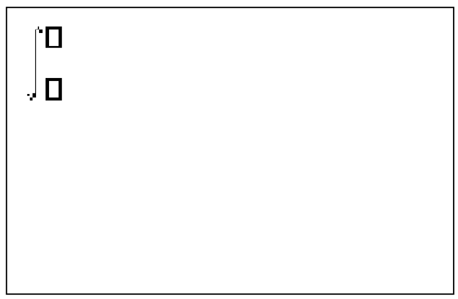
An example of a function and the region satisfying these conditions is shown in the figure below:



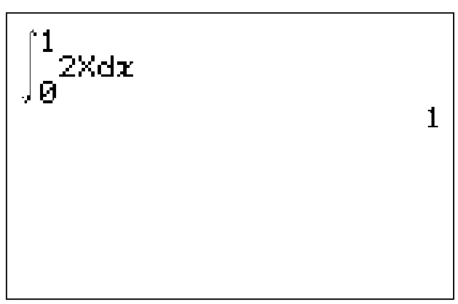
Calculator Operations

Prepare for the investigations in this chapter by setting the calculator to radian measure by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touching **B DRG**, and double **touching 2 Rad**. Set the calculator to floating point decimal display by touching **C FSE** and double touch **1 FloatPt**. Set the calculator to rectangular graphing by touching **E COORD**, and double touching **1 Rect**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the SET UP menu. Let's use the calculator to find an estimate of $\int_0^1 2x\,dx$.

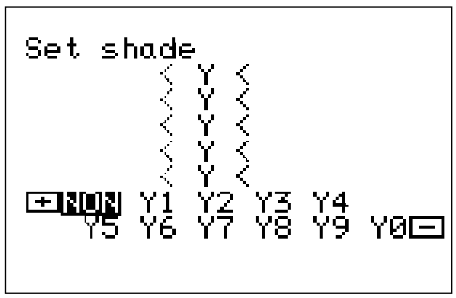
Access the numerical integrate function by pressing $\boxed{\substack{+\,-\\ \times\,\div}}$ $\boxed{\text{CL}}$ $\boxed{\text{MATH}}$, touch **A CALC**, and double touch **06** $\int$ to see:



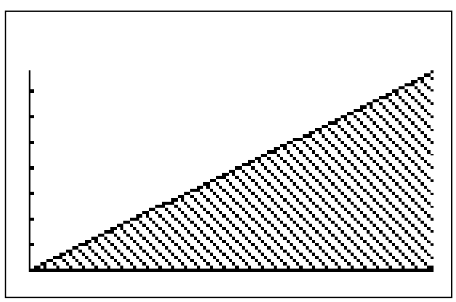
The blinking cursor is on the lower box asking for input of the lower limit of integration. Enter 0. Press $\boxed{\blacktriangle}$ and input 1 for the upper limit of integration by pressing $\boxed{1}$ $\boxed{\blacktriangleright}$. Next, press $\boxed{2}$ $\boxed{\text{X}/\theta/\text{T}/n}$ to input the integrand. The expression is incomplete and will result in an error message without the "*dx*", so press $\boxed{\text{MATH}}$ and double touch **07 dx**. Press $\boxed{\text{ENTER}}$ to view:



You can interpret this result geometrically by graphing the function over the indicated interval and shading the region whose area is represented by the integral. Press Y= CL to access and clear the Y1 prompt. Clear additional prompts by pressing ENTER CL . Enter $f(x)$ in Y1 with the keystrokes 2 X/θ/T/n . Press WINDOW and enter Xmin= 0 and Xmax= 1. Draw the graph by pressing ZOOM , touching **A ZOOM**, and double touching **1 Auto**. Next, to shade the region whose area is the value of the definite integral, press 2ndF DRAW , touch **G SHADE**, and double touch **1 Set** to access the shading screen:



Since Y1= 2X is the function "on the top," press ▶ to move to the upper bound, and touch **Y1** on the screen to place Y1 in the position. Since we are only dealing with one function, leave the lower bound location empty. Press GRAPH to view the shaded region:



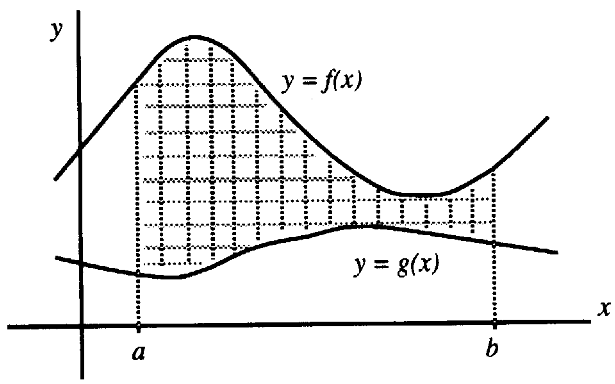
Notice that the area of the region between $f(x)= 2x$ and the x-axis from $x = 0$ to $x = 1$ is the area of the shaded triangle. This area equals 1/2 *base* • *height* = 1/2 (1)(2) = 1.

Therefore, the calculator's approximation to $\int_0^1 2x\, dx$ gives, in this instance, the exact value of the area. You will generally find the numerical approximation given by the calculator is fairly accurate for most functions you use in a beginning calculus course.

You can also use these ideas to find the area between two curves.

If f and g are continuous functions for $a \le x \le b$ and $f(x) \ge g(x)$ for $a \le x \le b$, the area of the region between $y = f(x)$, $y = g(x)$, and the vertical lines $x = b$ and $x = b$ is given by $\int_a^b f(x) - g(x)\, dx$.

An example of two functions and a region satisfying these conditions is shown in the figure below:



The area of the shaded region equals $\int_a^b f(x) - g(x)\, dx$.

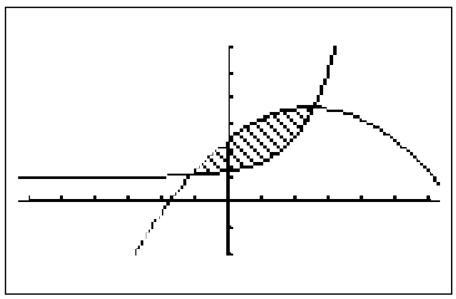
The area also equals $\int_a^b f(x)dx - \int_a^b g(x)dx$.

Students can choose either form of entry, but instructions in this chapter are given for the single integral.

Let's calculate and draw a graph of the area of the region between $f(x) = 5x - x^2 + 12$ and $g(x) = e^x + 5$. First, construct a graph to verify that the conditions for using an integral to calculate the area in this situation are satisfied. Press [2ndF] [DRAW], touch **G SHADE**, and double touch **2 INITIAL**. *You should do this before beginning each new problem*. Return to and clear the Y prompts by pressing [Y=] [CL]. Clear additional prompts if necessary. Input $f(x)$ in Y1 with the keystrokes [5] [X/θ/T/*n*] [–] [X/θ/T/*n*] [x^2] [+] [1] [2] [ENTER].

Input $g(x)$ in Y2 with the keystrokes 2ndF e^x X/θ/T/n ▶ + 5.
A little experimenting leads to a viewing window such as the one obtained with $^-6.3 \le x \le 6.3$, $^-10 \le y \le 30$. Your viewing window should clearly show the region between $f(x)$ and $g(x)$ and, if applicable, display the intersections of the functions. Enter the viewing window and press GRAPH to view the graphs.

Shade the region between the two curves by pressing 2ndF DRAW, touch **G SHADE**, and double touch **1 SET**. Touch Y2 since Y2 is the function "on the bottom," pressing ▶, and touch **Y1** since Y1 is the function "on the top." Press GRAPH to view the shaded region:



Next, find the limits of integration. Press 2ndF CALC and double touch **2 Intsct**. Do this twice to obtain the x-coordinates of the two points of intersection. You should obtain X = $^-1.09375$ and X = 2.583471.

Press + − × ÷ CL, press MATH, touch **A CALC**, double touch **06** ∫, enter $^-1.09375$ in the lower box, press ▲, enter 2.583471 in the upper box, press ▶, enter the function "on the top," $5x - x^2 + 12$, press – (, and then enter the function "on the bottom," $e^x + 5$. Close the parentheses by pressing) and complete the integral expression by pressing MATH and double touch **07 *dx***. Press ENTER to obtain the area 20.343776.

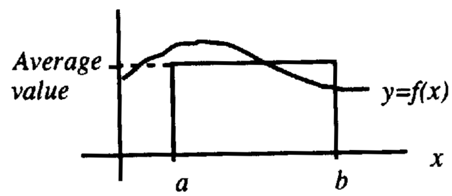
Method of Teaching

Use *Blackline Master 6.1* to create an overhead or handout for investigation of shading a region and calculating an area represented by an integral.
Use *Blackline Master 6.2* to create an overhead or handout for investigating shading the region between two curves and calculating its area.

Use *Blackline Master 6.3* to create an overhead or handout for investigating the average value of a function. Use the topics *For Discussion* to supplement the worksheets.

Using Blackline Master 6.3

In order to better explain the concept of *average value,* you can tell students that the average value of a non-negative continuous function $f(x)$ over the interval $a \le x \le b$ equals the height of the rectangle whose base is $b - a$ and whose area is the same as the area under the graph of f from a to b.



For Discussion

Explain to the students the concept of "negative" and "positive" area and discuss how this might affect calculations and shading.

Additional Problems

In problem 1-3, use the calculator's numerical integration function to obtain an approximation to the specified area. Draw a graph and shade the region whose area you have approximated:

1. the area bounded by $f(x)= x^3 - 2x$ and the x-axis for $2.4 < x < 5.8$.
2. the area bounded by $f(x)= e^{-x}$ and the x-axis for $^-2 < x < 1$.
3. the area between $f(x)= \sin x$, $g(x)= |\cos x|$, and the vertical line $x = \pi/4$ and $x = 3\pi/4$.
4. Find the average value of the function $f(x) = 1/x$ over the interval $1 \le x \le 5$.
5. Find $\int_{-1}^{3} x \, dx$.

Chapter seven

PROGRAMS FOR RECTANGULAR AND TRAPEZOIDAL APPROXIMATION OF AREA

Introducing the Topic

In this chapter, you and your students will review how to program the Sharp graphing calculator, execute a program, and use a program to calculate the area under a curve using rectangular and trapezoidal approximations. *Simpson's approximation* was used in the last chapter.

The RECTAPP program approximates the definite integral of a function using rectangles. When doing rectangle approximation, three estimates can be found. These are found by creating rectangles from the left endpoints of the given partition, right endpoints, and midpoint. The continuous function is entered within the program. The lower limit of the definite integral is entered as 'A,' the upper limit as 'B,' and the number of intervals in the partition as 'N.'

The TRAPAPP program approximates the definite integral of a function using trapezoids (trapezoid rule). Once again, the continuous function is entered within the program, the lower limit of the definite integral is entered as 'A,' the upper limit as 'B,' and the number of intervals in the partition as 'N.'

Calculator Operations

Turn the calculator on and set to radian mode by pressing [2ndF] [SET UP], touch **B DRG**, double touch **2 Rad**. Press [2ndF] [QUIT] to exit the SET UP menu. Press [2ndF] [PRGM] to enter programming mode. Touch **C NEW** to enter a new program, followed by [ENTER]. Name the new program RECTAPP by pressing [R] [E] [C] [T] [A] [P] [P] followed by [ENTER].

You can now enter in the RECTAPP program. Remember to press [ENTER] at the end of each line. If you make a mistake, use the calculator's editing features to make corrections. Enter the following program:

PROGRAM KEYSTROKES

(Input the number of intervals in the partition)

Input N	[2ndF] [PRGM] [A] [3] [ALPHA] [N] [ENTER]

(Input the lower limit of the definite integral)

Input A	[2ndF] [PRGM] [3] [ALPHA] [A] [ENTER]
Y1(A)⇒L	[VARS] [A] [ENTER] [A] [1] [(] [ALPHA] [A] [)] [STO] [ALPHA] [L] [ENTER]

(Input the upper limit of the definite integral)

Input B	[2ndF] [PRGM] [3] [ALPHA] [B] [ENTER]
Y1(B)⇒R	[VARS] [ENTER] [1] [(] [ALPHA] [B] [)] [STO] [ALPHA] [R] [ENTER]

(Compute the width of intervals in the partition)

(B–A)/N⇒W	[(] [ALPHA] [B] [–] [ALPHA] [A] [)] [÷] [ALPHA] [N] [STO] [ALPHA] [W] [ENTER]

(Compute first midpoint)

A+W/2⇒X	[ALPHA] [A] [+] [ALPHA] [W] [÷] [2] [STO] [X/θ/T/*n*] [ENTER]
Y1(X)⇒M	[VARS] [ENTER] [1] [(] [X/θ/T/*n*] [)] [STO] [ALPHA] [M] [ENTER]
A+W⇒X	[ALPHA] [A] [+] [ALPHA] [W] [STO] [X/θ/T/*n*] [ENTER]

(Calculate the approximations)

Label LOOP	[2ndF] [PRGM] [B] [1] [2ndF] [A-LOCK] [L] [O] [O] [P] [ALPHA] [ENTER]

Y1(X)+L⇒L	VARS ENTER 1 (X/θ/T/*n*) + ALPHA L STO ALPHA L ENTER
Y1(X)+R⇒R	VARS ENTER 1 (X/θ/T/*n*) + ALPHA R STO ALPHA R ENTER
X+W/2⇒X	X/θ/T/*n* + ALPHA W ÷ 2 STO X/θ/T/*n* ENTER
Y1(X)+M⇒M	VARS ENTER 1 (X/θ/T/*n*) + ALPHA M STO ALPHA M ENTER
X+W/2⇒X	X/θ/T/*n* + ALPHA W ÷ 2 STO X/θ/T/*n* ENTER
If X<BGoto	2ndF PRGM B 3 X/θ/T/*n* MATH F 5 ALPHA B
LOOP	2ndF PRGM B 2 2ndF A-LOCK L O O P ENTER
ClrT	2ndF PRGM C 1 ENTER
Print "LEFT="	2ndF PRGM A 1 2ndF PRGM 2 2ndF A-LOCK L E F T = 2ndF PRGM 2 ENTER
Print W*L	2ndF PRGM 1 ALPHA W × ALPHA L ENTER
Print "MID="	2ndF PRGM 1 2ndF PRGM 2 2ndF A-LOCK M I D = 2ndF PRGM 2 ENTER
Print W*M	2ndF PRGM 1 ALPHA W × ALPHA M ENTER
Print "RIGHT="	2ndF PRGM 1 2ndF PRGM 2 2ndF A-LOCK R I G H T = 2ndF PRGM 2 ENTER
Print W*R	2ndF PRGM 1 ALPHA W × ALPHA R ENTER
End	2ndF PRGM 6 ENTER

Press 2ndF QUIT to save the program and exit the editing mode.

Method of Teaching

Use *Blackline Masters 7.1* through *7.4* to create overheads and worksheets for entering and executing the RECTAPP and TRAPAPP programs. Go over in detail how to enter the program and what the different program lines are doing. Have the students enter the programs and execute them (correcting any errors).
Use the topics *For Discussion* to supplement the worksheets.

Using Blackline Masters 7.1-7.4

Enter the function you wish to approximate, say $f(x) = \cos x$, by pressing Y= CL cos X/θ/T/*n* . Press ENTER CL to clear additional prompts. To execute the RECTAPP program, press 2ndF PRGM A , highlight the 'RECTAPP' program, and press ENTER . You will be prompted to enter the number of intervals desired 'N,' followed by lower limit 'A' and upper limit 'B.' Enter 'N' of 5 by pressing 5 ENTER , followed by 'A' of 0 and 'B' of 1. You should then see the following display of left, midpoint and right rectangular approximations for the area under the cosine function between 0 and 1 with 5 intervals in the partition.

```
LEFT=
             .884633979
MID=
             .842875074
RIGHT=
              .79269444
```

You can repeat this process for a different number of partitions, different limits, or another function. Press ENTER to execute the program over and over again. Press CL to exit the program.

If you receive an error statement, press ◀ or ▶ to go to the line within the program in which the error occurs. Compare your line with the correct one above to find the error. Correct the error using the editing features of the calculator and save the program by pressing 2ndF QUIT . Try executing the program again.

Repeat the process to enter the following TRAPAPP program.

PROGRAM	KEYSTROKES
(Input the number of intervals in the partition)	
Input N	2ndF PRGM A 3 ALPHA N ENTER
(Input the lower limit of the definite integral)	
Input A	2ndF PRGM 3 ALPHA A ENTER
Y1(A)⇒L	VARS A ENTER A 1 (ALPHA A) STO ALPHA L ENTER
(Input the upper limit of the definite integral)	
Input B	2ndF PRGM 3 ALPHA B ENTER
Y1(B)⇒R	VARS ENTER 1 (ALPHA B) STO ALPHA R ENTER
0⇒Z	0 STO ALPHA Z ENTER
(Compute the width of intervals in the partition)	
(B–A)/N⇒W	(ALPHA B – ALPHA A) ÷ ALPHA N STO ALPHA W ENTER
A+W⇒X	ALPHA A + ALPHA W STO X/θ/T/*n* ENTER
(Calculate the approximation)	
Label LOOP	2ndF PRGM B 1 2ndF A-LOCK L O O P ALPHA ENTER
2Y1(X)+Z⇒Z	2 VARS ENTER 1 (X/θ/T/*n*) + ALPHA Z STO ALPHA Z ENTER
X+W⇒X	X/θ/T/*n* + ALPHA W STO X/θ/T/*n* ENTER
If X<BGoto	2ndF PRGM 3 X/θ/T/*n* MATH F 5 ALPHA B
LOOP	2ndF PRGM 2 2ndF A-LOCK L O O P ENTER
ClrT	2ndF PRGM C 1 ENTER
Print "TRAP="	2ndF PRGM A 1 2ndF PRGM 2 2ndF A-LOCK T R A P = 2ndF PRGM 2 ENTER
Print (W/2)	2ndF PRGM 1 (ALPHA W ÷ 2)
(L+Z+R)	(ALPHA L + ALPHA Z + ALPHA R) ENTER
End	2ndF PRGM 6 ENTER

Execution of the TRAPAPP program for an ‘n’ of 5, an ‘a’ of 0, and a ‘b’ of 1 should result in the trapezoidal approximation of .838664209 for the area under the cosine function between 0 and 1 with 5 intervals in the partition.

For Discussion

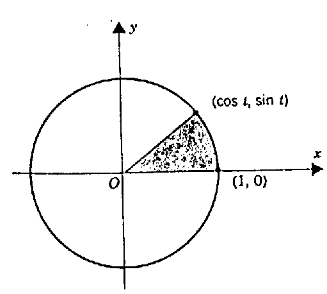
You and your students can discuss how you can change the functions, the relationships between the approximations, and the changes in the approximations when the number of intervals is increased for a fixed function and interval.

Chapter eight

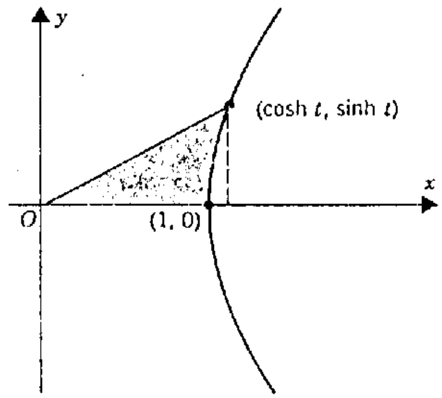
HYPERBOLIC FUNCTIONS

Introducing the Topic

In this chapter, you and your students will learn how to graph and evaluate hyperbolic functions on the Sharp EL-9600. The hyperbolic functions are defined the same as trigonometric functions, except the hyperbolic functions use a hyperbola instead of a circle. In trigonometry, the coordinates of any point on the unit circle can be defined $(x, y) = (\cos t, \sin t)$ where t is the measure of the arc from (x, y) to $(1, 0)$. In radians, the central angle = t and twice the area of the shaded region equals t.



Whereas, with hyperbolic functions the coordinates of any point on the unit hyperbola can be defined as $(x, y) = (\cosh t, \sinh t)$. Once again, t is the measure of the arc from (x, y) to $(1, 0)$ and t equals twice the area of the shaded region.

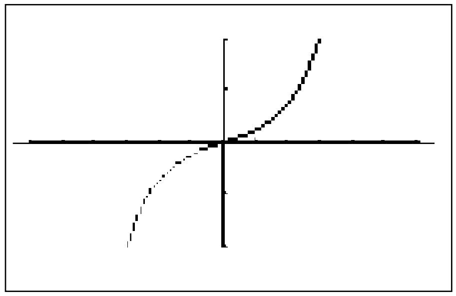


The four additional hyperbolic functions are defined in terms of hyperbolic sine and hyperbolic cosine. The six hyperbolic functions are expressed as y = sinh x, y = cosh x, y = tanh x, y = csch x, y = sech x, and y = coth x.

Calculator Operations

Turn the calculator on and press [Y=] [CL] to access and clear the Y1 prompt. Press [ENTER] [CL] to remove additional expressions. The calculator should be setup in radian measurement with rectangular coordinates. To complete this setup, press [2ndF] [SET UP] touch **B DRG**, double touch **2 Rad**, touch **E COORD**, and double touch **1 Rect**. Press [2nd] [QUIT] to exit the SET UP menu and return to the Y prompts.

To enter the hyperbolic sine function (y = sinh x) for Y1, press [MATH] , touch **A CALC**, touch ▼ on the screen, double touch **15 sinh**, and press [X/θ/T/*n*]. Enter the viewing window by pressing [ZOOM], touching **F HYP**, and double touching **1 sinh *x***.



The remaining five hyperbolic functions can be graphed in a similar manner.

Method of Teaching

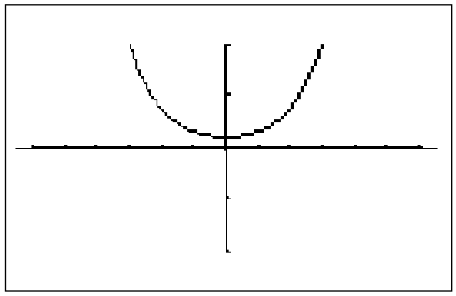
Use the *Blackline Master 8.1* to create an overhead for graphing the hyperbolic sine function. Go over in detail how hyperbolic functions are derived and how to graph them. Follow the hyperbolic sine function demonstration with the hyperbolic cosine demonstration discussed in Using *Blackline Master 8.1* and appearing on *Blackline Master 8.1*.

Next, use the *Blackline Masters 8.2* and *8.3* to create worksheets for the students. Have the students graph the remaining four hyperbolic functions. Use the topics *For Discussion* to supplement the worksheets.

Using Blackline Master 8.1

The problem discussed above under Calculator Operations is presented on *Blackline Master 8.1*. The following problem also appears on the *Blackline Master*. Graph the hyperbolic cosine function by pressing $\boxed{\text{Y=}}$ $\boxed{\text{CL}}$ to remove the hyperbolic sine function.

To enter the hyperbolic cosine function ($y = \cosh x$) for Y1, press $\boxed{\text{MATH}}$, touch ⏷ on the screen, double touch **16 cosh *x***, press $\boxed{\text{X}/\theta/\text{T}/n}$. Use the same viewing window as before ($^-6.5 < x < 6.5$ by $^-10 < y < 10$) and press $\boxed{\text{GRAPH}}$ to view the graph.

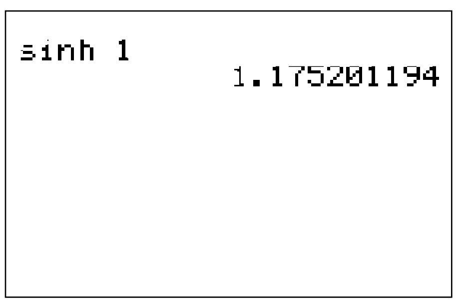


For Discussion

You and your students can discuss how the hyperbolic functions differ from the trigonometric functions and why. Where do the hyperbolic functions increase and decrease? What are their domain and ranges? Does periodicity exist? Why or why not?

EVALUATION OF HYPERBOLIC FUNCTIONS

Evaluating hyperbolic functions is easy on the Sharp EL-9600 graphing calculator. First, access the home or computational screen by pressing [+ − × ÷] [CL] . Second, enter the expression to be evaluated, for example sinh 1. Enter sinh 1 by pressing [MATH] , touching ▼ on the screen, double touching **15 sinh**, and pressing [1] [ENTER] .



Chapter nine

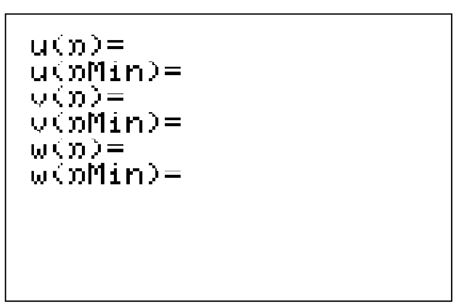
SEQUENCES AND SERIES

Introducing the Topic

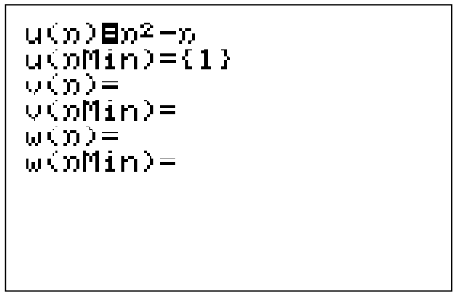
In this chapter, you and your students will learn how to create a table for and graph of a sequence. You will also examine recursive sequences, including the Fibonnoci sequence. In addition, you will learn how to calculate the partial sum of a series.

Calculator Operations

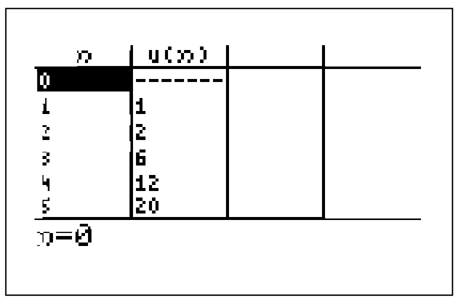
Turn the calculator on and set the calculator to sequence mode by pressing 2ndF SET UP , touching **E COORD**, and double touching **4 Seq**. Press 2ndF QUIT Y= to access the sequence prompts. Clear any sequences by pressing CL .



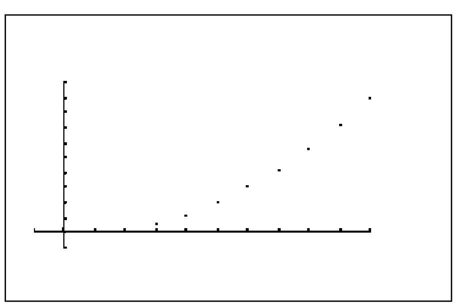
Enter the sequence generator $a_n = n^2 - n$ for $u(n)$ by pressing X/θ/T/*n* x^2 – X/θ/T/*n* ENTER . Enter $n_1 = 1$ for $u(n\text{Min})$ by pressing 1 ENTER .



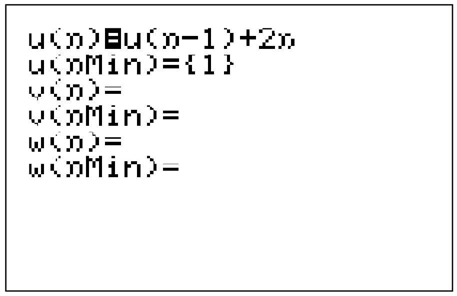
View a table of sequence values by pressing TABLE .



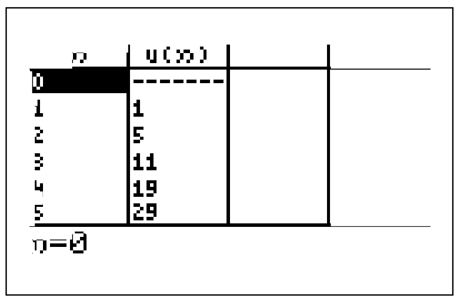
Graph the sequence by first setting the format to time and dot modes. Do this by pressing 2ndF FORMAT , touching **G TYPE**, double touching **2 Time**, touching **E STYLE1**, and double touching **2 Dot**. Press 2ndF QUIT to exit the FORMAT menu. Enter the viewing window by pressing WINDOW EZ , double touching **3**, double touching **5 -1<X<10**, and double touching **2 -10<Y<100**.



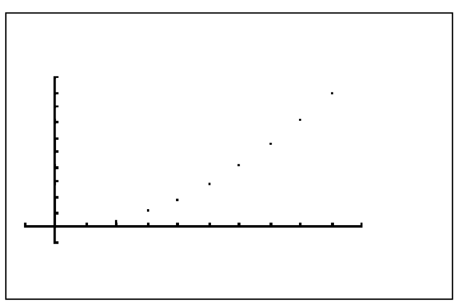
Enter the recursive sequence generator $a_n = a_n - 1 + 2n$ for $u(n)$ by pressing
$\boxed{\text{Y=}}$ $\boxed{\text{CL}}$ $\boxed{\text{2ndF}}$ $\boxed{\text{u}}$ $\boxed{(}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{-}$ $\boxed{1}$ $\boxed{)}$ $\boxed{+}$ $\boxed{2}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{\text{ENTER}}$.
Enter $a_1 = 1$ by pressing $\boxed{1}$ $\boxed{\text{ENTER}}$.



View a table of sequence values by pressing $\boxed{\text{TABLE}}$.



Graph the sequence in the same window as above by pressing $\boxed{\text{GRAPH}}$.

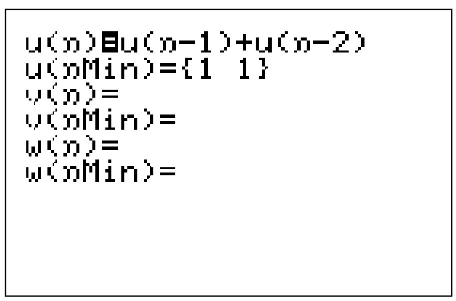


Method of Teaching

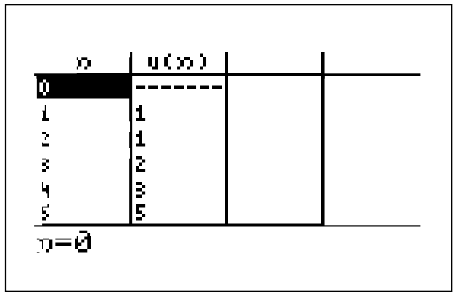
Use *Blackline Master 9.1* to create an overhead or worksheet for entering a sequence, generating a table for the sequence, and graphing the sequence. Use *Blackline Master 9.2* to create an overhead or worksheet for entering a recursive sequence, generating a table, and graphing. Use *Blackline Master 9.3* to create a worksheet on entering the Fibonocci sequence, making a table for it, and graphing it. Use the topics *For Discussion* to supplement the worksheets.

Using Blackline Master 9.3

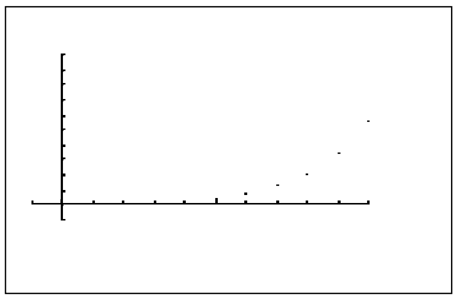
The Fibonnoci sequence is the sequence where the previous two terms are added together to form the next term. The first two terms of the sequence are $a1 = 1$ and $a2 = 1$. Enter the Fibonnoci sequence generator $a_n = a_{n-1} + a_{n-2}$ for $u(n)$ by pressing $\boxed{\text{Y=}}$ $\boxed{\text{CL}}$ $\boxed{\text{2ndF}}$ $\boxed{\text{u}}$ $\boxed{(}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{-}$ $\boxed{1}$ $\boxed{)}$ $\boxed{+}$ $\boxed{\text{2ndF}}$ $\boxed{\text{u}}$ $\boxed{(}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{-}$ $\boxed{2}$ $\boxed{)}$ $\boxed{\text{ENTER}}$. Enter $a_1 = 1$ and $a_2 = 1$ by pressing $\boxed{\text{2ndF}}$ $\boxed{\{}$ $\boxed{1}$ $\boxed{,}$ $\boxed{1}$ $\boxed{\text{2ndF}}$ $\boxed{\}}$ $\boxed{\text{ENTER}}$.



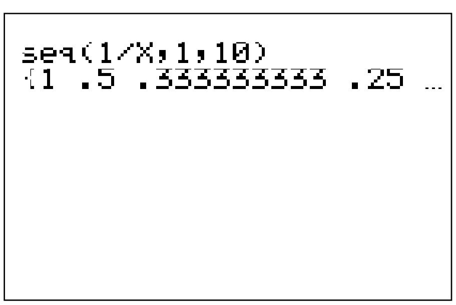
Notice that upon entry, the comma disappears from the sequence screen. View a table of sequence values by pressing $\boxed{\text{TABLE}}$.

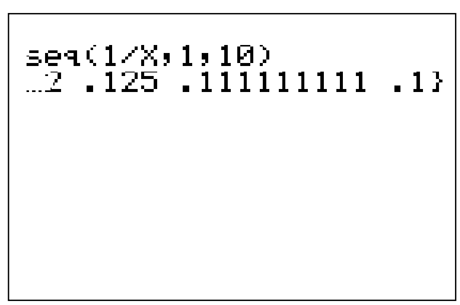


Graph the sequence in the same window as before by pressing GRAPH.

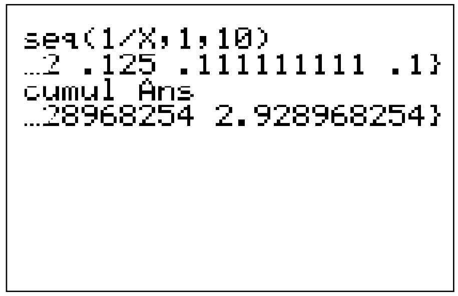


Find the partial sum of a series, ∑ 1/X, for the first 10 terms. Return the SET UP to rectangular mode by pressing 2ndF SET UP, touching **E COORD**, and double touching **1 Rect**. Press 2ndF QUIT to exit the SET UP menu. First find the sequence of the first 10 terms of series ∑ 1/X by first pressing CL 2ndF LIST, touching **A OPE**, and double touching **5 seq(** . Enter the generator 1/X by pressing 1 ÷ X/θ/T/*n* ,. Enter the lower and upper bounds for the sequence by pressing 1 , 1 0). Find the sequence by pressing ENTER. Press ▶ to see more of the sequence.





Find the partial sum of the first 10 terms by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{LIST}}$, double touching **6 cumul**, and pressing $\boxed{\text{2ndF}}$ $\boxed{\text{ANS}}$ $\boxed{\text{ENTER}}$. Press $\boxed{\blacktriangleright}$ to move right in the sequence of partial sums until the last term is seen. This is the partial sum of the first 10 terms.



For Discussion

You and your students can discuss how to store a sequence into a list, and perform other calculations of the list.

Chapter ten

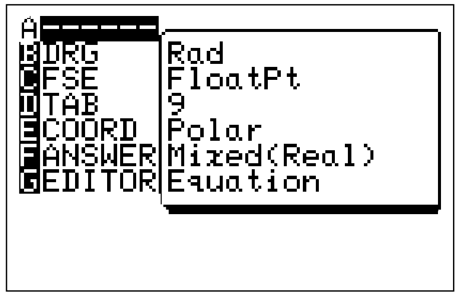
GRAPHING PARAMETRIC AND POLAR EQUATIONS

Introducing the Topic

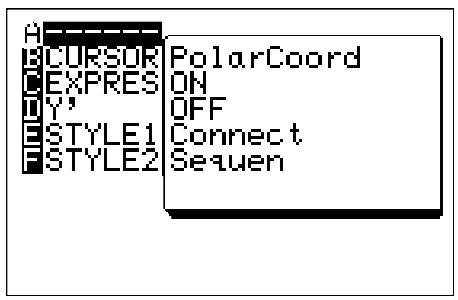
In this chapter, you and your students will learn how to graph polar ($R = f(\theta)$) and parametric equations on the Sharp EL-9600. The ordered pairs of polar functions are defined as a 'R' (radius, distance from origin) in terms of an angle measurement in radians (measured from the initial ray beginning at the origin and passing through (1,0)). Whereas, the ordered pairs of a parametric function are defined as 'X' and 'Y,' but both 'X' and 'Y' are in terms of another variable 'T' (usually time).

Calculator Operations

Turn the calculator on and press $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touch **E COORD**, and double touch **3 Polar** to change to polar mode. While in the SET UP menu, the calculator should be setup in radian mode. To complete this setup, touch **B DRG**, and double touch **2 Rad**.

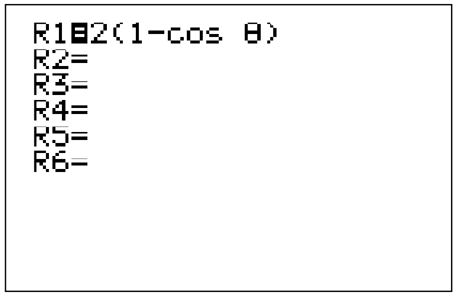


Press 2ndF QUIT to exit the SET UP menu. Make sure calculator is in connected mode by pressing 2ndF FORMAT , touching **E STYLE1**, and double touching **1 Connect**. While in the FORMAT menuset the calculator to display polar coordinates when tracing by touching **B CURSOR** and double touching **2 PolarCoord**. Also, set the calculator to display the expression during tracing by touching **C EXPRESS** and double touching **1 ON**.

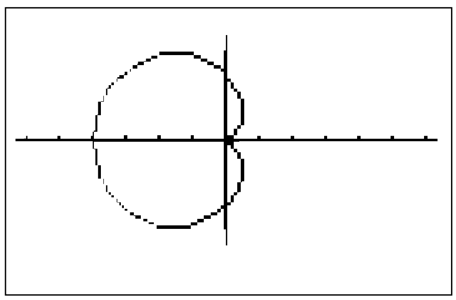


Press 2ndF QUIT to exit the FORMAT menu. Press Y= to access the R1 prompt. Press CL to remove an old R1 expression. Press ENTER CL to clear additional R prompts.

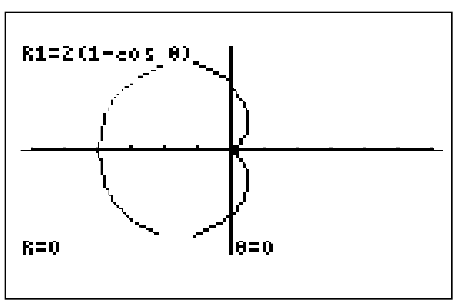
Enter the polar function $r = 2(1 - \cos \theta)$ for R1, by pressing 2 (1 – cos X/θ/T/*n*) . Notice, when in polar mode the X/θ/T/*n* key provides a θ for equation entry.



Now, graph the polar function in the Decimal viewing window by pressing ZOOM , touching **A ZOOM**, touching ▼ on the screen, and touching **7 Dec**.



This particular shape of curve is called a cardoid. Trace the curve by pressing TRACE . Notice the expression is displayed at the top of the screen.

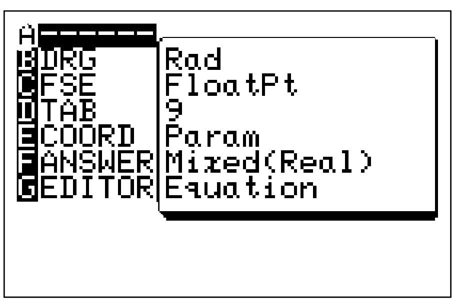


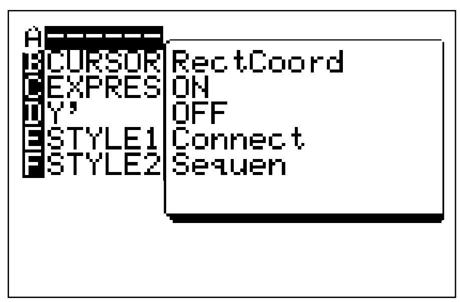
Method of Teaching

Use *Blackline Masters 10.1* and *10.2* to create overheads for graphing polar and parametric equations. Go over in detail how to change modes, adjust the format, and graph equations. Use *Blackline Masters 10.3* and *10.4* to create a worksheets for the students. Have the students graph the four equations (2 polar, 2 parametric). Use the topics *For Discussion* to supplement the worksheets. Engage the trace feature by pressing TRACE, followed by ◀ and ▶. Tracing will assist you in discussing the topics.

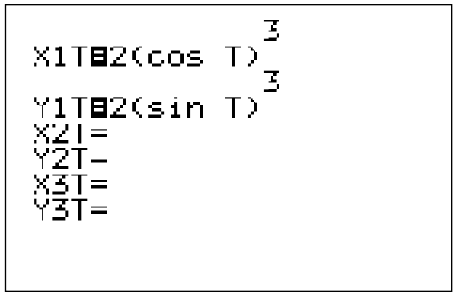
Using Blackline Master 10.2

The problem discussed above under *Calculator Operations* is presented on *Blackline Master 10.1*. The following discussion appears on *Blackline Master 10.2*. Turn the calculator on and press 2ndF SET UP, touch **E COORD**, and double touch **2 Param** to change to parametric mode. Press 2ndF QUIT to exit the SET UP menu. Make sure calculator is set to display rectangular coordinates when tracing by pressing 2ndF FORMAT, touching **B CURSOR** and double touching **1 RectCoord**. Press 2ndF QUIT to exit the FORMAT menu.

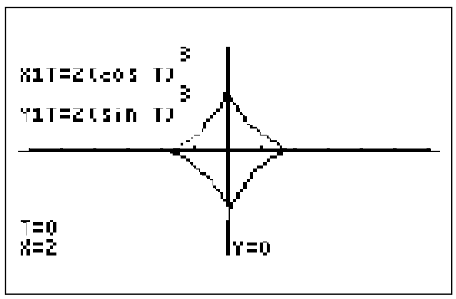




To enter the parametric function X1T = $2(\cos T)^3$, Y1T = $2(\sin T)^3$, press Y= CL 2 (cos X/θ/T/*n*) a^b 3 ENTER CL 2 (sin X/θ/T/*n*) a^b 3 ENTER. Notice, when in parametric mode the X/θ/T/*n* key provides a T for equation entry.



Now, graph the parametric function in the Decimal viewing window by pressing ZOOM , touching **A ZOOM**, touching ▼ on the screen, and touching **7 Dec**. Press **TRACE** and notice the expression and T values now appear on the range screen.



For Discussion

You and your students can use the calculator to discuss how functions are generated from t = 0 or θ = 0 through their maximum (use the trace feature).

Additional Problems

Graph the following equations:

1. R1 = 2sin (3θ) (three leaved rose or propeller)
   Decimal viewing window.

2. R1 = 3 sin θ (circle)
   Decimal viewing window.

3. X1T = sin T
   Y1T = cos (2T)
   Default viewing window.

4. X1T = 3T2
   Y1T = T3 – 3T
   $^{-}\sqrt{3} < t < \sqrt{3}$, $0 < x < 12.6$, $^{-}3.1 < y < 3.1$

CONTENTS OF REPRODUCIBLE BLACKLINE MASTERS

Use these reproducible Blackline Masters to create handouts, overhead transparencies, and activity worksheets.

NAME ____________________ CLASS __________ DATE __________

1.1

EVALUATING LIMITS GRAPHICALLY

1. Set the calculator to floating point decimal display by pressing [2ndF] [SET UP], touching **C FSE**, and double touching **1 Float Pt**. Set the calculator to rectangular coordinates by touching **E COORD** and double touching **1 Rect**. Press [2ndF] [QUIT] to exit the set up menu.

2. Consider the function $f(x) = \dfrac{(2x + 2)}{(x^2 - 1)}$. Press [Y=] [CL] to access and clear the Y1 prompt. Press [ENTER] [CL] to clear the remaining prompts.

3. Construct a graph of $f(x)$ in the Decimal viewing window by first entering Y1= $\dfrac{(2x + 2)}{(x^2 - 1)}$ with the keystrokes [a/b] [2] [X/θ/T/*n*] [+] [2] [▶] [X/θ/T/*n*] [x^2] [–] [1], and then press [ZOOM], touch **A ZOOM**, touch ▼ on the screen, and touch **7 Dec** to see the graph.

4. Notice that even though $\dfrac{(2x + 2)}{(x^2 - 1)}$ is not defined at $x = {}^-1$ (as evidenced by the hole in the graph at that point), the functional values appear to be getting closer and closer to ⁻1. A careful observation of the graph leads to the following estimates:

   $\lim_{x \to -\infty} f(x) = 0,$

   $\lim_{x \to {}^-1} f(x) = {}^-1,$

   $\lim_{x \to 1} f(x)$ does not exist,

   and $\lim_{x \to \infty} f(x) = 0.$

5. It also appears that the line $y = 0$ is a horizontal asymptote and the line $x = 1$ is a vertical asymptote for this function.

NAME ______________________________ CLASS __________ DATE __________

1.2

EVALUATING LIMITS NUMERICALLY

1. Tables of functional values sometimes provide more detailed information than a graph when investigating limits. The Sharp EL-9600 has a TABLE feature to assist you in constructing a numerical table of values. Press $\boxed{\text{TABLE}}$ to access the TABLE feature.

X	Y1		
0	-2		
1	-------		
2	2		
3	1		
4	.666667		
5	.5		

X=0

2. Notice the table provides the *x* values and their corresponding *y*-values according to Y1. You can change the table settings by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{TBLSET}}$. You can change the table start value and the table step value. Verify the following values using the the TABLE feature.

x gets smaller and smaller→

x	¯10	¯50	¯100	¯250	¯500	¯1000	¯10,000
y	¯.18182	¯.03922	¯.01980	.00797	¯.00399	¯.00200	¯.00020

y = *f*(*x*) appears to get closer and closer to 0

This provides evidence that $\lim_{x \to \infty} f(x) = 0$.

x approaches -1 from the left | *x* approaches -1 from the right

x	¯1.05	1.01	¯1.001	¯1.0001		¯.9999	¯.999	¯.99	¯.90
y	¯.9756	¯.99502	¯.9995	¯.9999		¯1.0001	¯1.0005	¯1.0051	¯1.0526

y = *f*(*x*) gets closer and closer to ¯1 from above | *y* = *f*(*x*) gets closer and closer to ¯1 from below

This provides evidence that $\lim_{x \to {}^{-}1} f(x) = {}^{-}1$.

NAME ______________________________ CLASS __________ DATE __________

1.3

EVALUATING LIMITS

1. Consider the graph of $f(x) = \dfrac{|(x+1)|}{(x+1)}$ in a decimal viewing window. Trace the graph for values of x less than $x = {}^-1$. Describe the values of $f(x)$ for $x < {}^-1$.

   ____

   Describe the values of $y = f(x)$ for $x > {}^-1$.

   ____

   Do you feel this behavior of the function continues for all values of x; that is, as x gets smaller and smaller and as x gets larger and larger?

   ____

2. Use the TABLE feature with $f(x) = \dfrac{(x^2 + 2x - 3)}{(x^2 - 9)}$ to estimate the following limits.

   $\lim_{x \to {}^-\infty} f(x) =$ __________

   $\lim_{x \to {}^-3} f(x) =$ __________

   $\lim_{x \to 3} f(x) =$ __________

   $\lim_{x \to \infty} f(x) =$ __________

3. The population of fish in a certain small lake at time t months after it is stocked is given by $p(t) = \dfrac{10{,}000}{(1 + 15e^{-t/4})}$. Due to a limited supply of food and oxygen, the growth of the population is limited. Find the size of the population 20 days after the lake is stocked and the limiting value of the size of the population of fish.

NAME ______________________________ CLASS __________ DATE __________

2.1

DERIVATIVES USING THE LIMIT DEFINITION

Activity 1

1. Set the calculator to radian measure by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touching **B DRG**, and double touching **2 Rad**. Set the calculator to floating point decimal display by touching **C FSE** and double touching **1 Float Pt**. Set the calculator to rectangular coordinates by touching **E COORD** and double touching **1 Rec**t. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the SET UP menu.

2. Consider the function $f(x) = x^2$ at $x = 2$ where $f'(2) =$ $\lim_{x\to 2} \frac{[f(x) - f(2)]}{(x-2)} = \lim_{x\to 2} \frac{(x^2-4)}{(x-2)}$.

3. Enter Y1$=\frac{(x^2-4)}{(x-2)}$ and use the TABLE feature to verify the following table.

x	1.9	1.99	1.999	1.9999			2.0001	2.001	2.01	2.1
y	3.9	3.99	3.999	3.9999			4.0001	4.001	4.01	4.1

Activity 2

1. Consider the function $f(x) = 2x^3$ at $x = 1$ where $f'(1)$ equals $\lim_{x\to 1} \frac{[f(x) - f(1)]}{(x-1)} = \lim_{x\to 1} \frac{(2x^3-2)}{(x-1)}$.

2. Enter Y1 $= \frac{(2x^3-2)}{(x-1)}$ and use the TABLE feature to build the following table.

x	.9	.99	.999	.9999			1.0001	1.001	1.01	1.1
y										

It appears that $\lim_{x\to 1} \frac{(2x^3-2)}{(x-1)} = f'(1) =$.

Activity 3

1. Consider the function $f(x) = 2x^3$ at $x = {}^-3$. $f'({}^-3)=$_____

NAME ____ CLASS __________ DATE __________

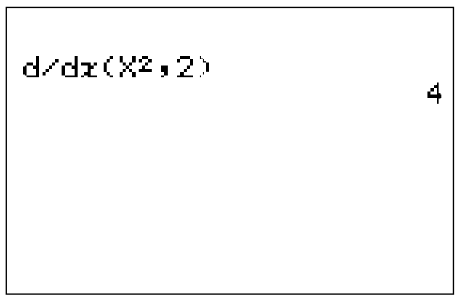
2.2

DERIVATIVES USING THE d/dx FUNCTION

Activity 1

1. The calculator has a built-in function denoted by d/dx that uses numerical methods to estimate the derivative of a function at a given value. The entry form of the derivative function is $d/dx\ (f(x), a)$.

2. Estimate $f'(2)$ for $f(x) = x^2$ using the derivative function. Press $\boxed{+\,-\,\times\,\div}$ $\boxed{\text{MATH}}$, touch **A CALC**, double touch **05 *d/dx*(**, press $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{x^2}$ $\boxed{,}$ $\boxed{2}$ $\boxed{)}$ to input: $d/dx\ (X^2, 2)$. Press $\boxed{\text{ENTER}}$ to compute.



Activity 2

1. Estimate $f'(1)$ for $f(x) = 2x^3$ using the derivative function.

2. $f'(1)$ = __________

Activity 3

1. Investigate $f'(0)$ for $f(x) = |x|$ using the derivative function. Complete the following table.

x	⁻.1	.01	⁻.001	⁻.0001		.0001	.001	.01	.1
$f'(x)$									

What is your conclusion regarding $f'(0)$?

____

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2.3

DERIVATIVES USING THE DERIVATIVES TRACE

Activity 1

1. Activate the derivative trace by pressing [2ndF] [FORMAT], touch **D Y'**, and double touch **1 ON**. Press [2ndF] [QUIT] to exit the **FORMAT** menu.

2. Press [Y=] [CL] and enter the function for Y1 by pressing [X/θ/T/*n*] [x^2].

3. Draw the graph by pressing [WINDOW] [EZ], touching **5**, touching ▼ on the screen, double touching **7 -5<*x*<5**, and touching **4 -10<*Y*<10**.

4. Press [TRACE] to activate the trace and press [▶] to the values for the derivative.

Activity 2

1. Enter $f(x)$ for Y1 and construct the graph by pressing [ZOOM], touching **A ZOOM**, touching ▼ on the screen, and touching **7 Dec**. Press [ZOOM], touching **A ZOOM**, and double touching **1 Auto**.

2. Complete the following tables of values. Use the pattern you view in the table of values to determine a formula for $f'(x)$.

a. $f(x) = e^{2x}$

x	y	y'
$^-3$		
$^-2$		
$^-1$		
0		
1		
2		
3		

b. $f(x) = 2.5\,x^2$

x	y	y'
$^-3$		
$^-2$		
$^-1$		
0		
1		
2		
3		

Remember to turn off the derivative trace.

NAME ______________________________ CLASS ________ DATE ________

3.1

TANGENT LINES

Enter and execute a program for finding a tangent line to a curve at a given point.

1. Turn the calculator on and press $\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ to enter the programming menu.

2. Touch **C NEW** and press $\boxed{\text{ENTER}}$ to open a new program.

3. Name the new program TANGENT by pressing $\boxed{\text{T}}$ $\boxed{\text{A}}$ $\boxed{\text{N}}$ $\boxed{\text{G}}$ $\boxed{\text{E}}$ $\boxed{\text{N}}$ $\boxed{\text{T}}$ $\boxed{\text{ENTER}}$.

4. Press $\boxed{\text{ENTER}}$ at the end of each line. If you make a mistake, use the calculator's editing features to correct the error. Enter the following program:

PROGRAM	KEYSTROKES
Input X	$\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{\text{A}}$ $\boxed{\text{3}}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{\text{ENTER}}$
$\frac{d}{dx}$(Y1,X)⇒M	$\boxed{\text{MATH}}$ $\boxed{\text{A}}$ $\boxed{\text{0}}$ $\boxed{\text{5}}$ $\boxed{\text{VARS}}$ $\boxed{\text{A}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{A}}$ $\boxed{\text{1}}$ $\boxed{,}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{)}$
	$\boxed{\text{STO}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{M}}$ $\boxed{\text{ENTER}}$
Y1(X)⇒Y	$\boxed{\text{VARS}}$ $\boxed{\text{A}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{1}}$ $\boxed{(}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{)}$ $\boxed{\text{STO}}$ $\boxed{\text{ALPHA}}$
	$\boxed{\text{Y}}$ $\boxed{\text{ENTER}}$
⁻M•X+Y⇒B	$\boxed{(-)}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{M}}$ $\boxed{\times}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{+}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{Y}}$ $\boxed{\text{STO}}$ $\boxed{\text{ALPHA}}$
	$\boxed{\text{B}}$ $\boxed{\text{ENTER}}$
ClrT	$\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{\text{C}}$ $\boxed{\text{1}}$ $\boxed{\text{ENTER}}$
Print	$\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{\text{A}}$ $\boxed{\text{1}}$ $\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{\text{2}}$ $\boxed{\text{2ndF}}$
"TANGENT	$\boxed{\text{ALPHA}}$ $\boxed{\text{T}}$ $\boxed{\text{A}}$ $\boxed{\text{N}}$ $\boxed{\text{G}}$ $\boxed{\text{E}}$ $\boxed{\text{N}}$ $\boxed{\text{T}}$ $\boxed{\text{SPACE}}$
LINE=	$\boxed{\text{L}}$ $\boxed{\text{I}}$ $\boxed{\text{N}}$ $\boxed{\text{E}}$ $\boxed{=}$ $\boxed{\text{ENTER}}$
Print	$\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{\text{1}}$ $\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{\text{2}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{Y}}$ $\boxed{\text{ALPHA}}$
"Y=MX+B	$\boxed{=}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{M}}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{+}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$ $\boxed{\text{ENTER}}$

NAME ______________________ CLASS __________ DATE __________

3.2

TANGENT LINES

Continue to enter the TANGENT program.

Print "M=	[2ndF] [PRGM] [1] [2ndF] [PRGM] [2] [ALPHA] [M] [ALPHA] [=] [ENTER]
Print M	[2ndF] [PRGM] [1] [ALPHA] [M] [ENTER]
Print "B=	[2ndF] [PRGM] [1] [2ndF] [PRGM] [2] [ALPHA] [B] [ALPHA] [=] [ENTER]
Print B	[2ndF] [PRGM] [1] [ALPHA] [B] [ENTER]
End	[2ndF] [PRGM] [6] [ENTER]

5. Press [2ndF] [QUIT] to save the program and exit the editing mode.

6. Enter $f(x) = x^3 - x^2 + 1$ for Y1 by pressing [Y=] [CL] [X/θ/T/*n*] [a^b] [3] [▶] [–] [X/θ/T/*n*] [x^2] [+] [1] [ENTER] .

7. Execute the TANGENT program by pressing [2ndF] [PRGM] , touching **A EXEC**, and double touching **TANGENT**.

8. Enter an X of 1 by pressing [1] [ENTER] . You should then see the equation for the tangent line to the curve at $x = 1$.

9. You can repeat this process for other x values. Press [ENTER] to execute the program over and over again.

10. If you receive an error statement, press [◀] or [▶] to go to the line within the program in which the error occurs. Compare your line with the correct one above to find the error. Correct the error using the editing features of the calculator and save the program by pressing [2ndF] [QUIT] . Try executing the program again.

NAME ______________________________ CLASS __________ DATE __________

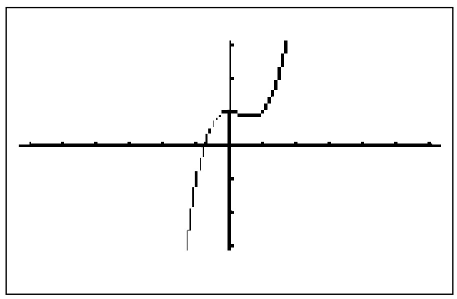
3.3

TANGENT LINES

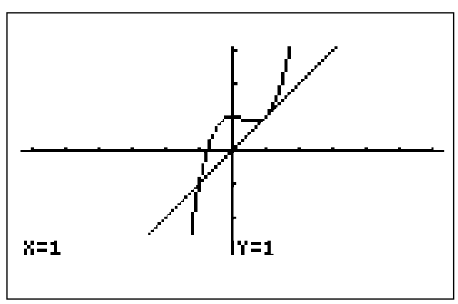
Draw the tangent line to a function at a given point.

1. To draw the tangent line on a displayed graph, you must first enter the graph for Y1. Enter the function $f(x) = x^3 - x^2 + 1$ for Y1 by pressing [Y=] [CL] [X/θ/T/*n*] [a^b] [3] [▶] [–] [X/θ/T/*n*] [x^2] [+] [1] [ENTER] .

2. Graph the function by pressing [ZOOM] , touching **A ZOOM**, touching ⏷ on the screen, and touching **7 Dec**.



3. Draw the tangent line at $x = 1$ by pressing [2ndF] [DRAW] , touching **A DRAW**, double touching **5 T_line(**, move the tracer right to $x = 1$ by pressing [▶] repeatedly, and then press [ENTER] .



NAME ______________________________ CLASS __________ DATE __________

4.1

DERIVATIVES

1. Set the graphing calculator to radian measure, floating point decimal display, and rectangular graphing by pressing [2ndF] [SET UP], touching **B DRG**, double touching **2 Rad**, touching **C FSE**, double touch **1 FloatPt**, touching **E COORD**, and double touching **1 Rect**. Press [2ndF] [QUIT] to exit the SET UP menu.

2. Choose sequential graphing and connected mode by pressing [2ndF] [FORMAT], touching **F STYLE2**, double touching **1 Sequen**, touching **E STYLE1** and double touching **1 Connect**. Press [2ndF] [QUIT] to exit the FORMAT menu.

3. Consider $f(x) = 2x^3 - 7x^2 - 70x + 75$. Press [Y=] [CL] to access and clear Y1. Press [ENTER] [CL] to clear additional Y prompts. Enter $f(x)$ for Y1 by pressing [2] [X/θ/T/n] [a^b] [3] [▶] [–] [7] [X/θ/T/n] [x^2] [–] [7] [0] [X/θ/T/n] [+] [7] [5].

4. Graph the function by pressing [WINDOW] [EZ], touching **5**, double touching **5 ⁻10 < X < 10**, and double touching **1 ⁻500 < Y < 500**.

5. To determine the point at which the relative maximum occurs, press [TRACE] [2ndF] [CALC], and double touch **4 Maximum**.

6. Find the point at which the relative minimum occurs by pressing [2ndF] [CALC], and double touch **3 Minimum**.

7. Combining this information with a view of the graph, we see that $f(x)$ is increasing from $^-\infty$ to $^-2.4427$ and from 4.776 to ∞ while $f(x)$ is decreasing for x between $^-2.4427$ and 4.776.

NAME ______________________________ CLASS __________ DATE __________

4.2

GRAPHS OF DERIVATIVES

1. Graph $f'(x)$ by pressing $\boxed{\text{Y=}}$ $\boxed{\text{ENTER}}$ and input $\frac{d}{dx}$ (Y1) in the Y2 location with the keystrokes $\boxed{\text{MATH}}$, touch **A CALC**, double touch **05 d/dx(** , press $\boxed{\text{VARS}}$ $\boxed{\text{ENTER}}$, touch **A XY**, double touch **1 Y1**, and press $\boxed{)}$ $\boxed{\text{ENTER}}$. Press $\boxed{\text{GRAPH}}$ to obtain the graphs of $f(x)$ and $f'(x)$.

2. We now want to find the two x-intercepts of $f'(x)$. Press $\boxed{\text{TRACE}}$ $\boxed{\blacktriangledown}$ to place the tracer on the graph of the derivative. Then, press $\boxed{\text{2ndF}}$ $\boxed{\text{CALC}}$ and double touch **5 X_Incpt**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{CALC}}$ and double touch **5 X_Incpt** again to obtain the other x-intercept.

3. Comparing these values to the x-coordinates of the points at which the maxima and minima of $f(x)$ occur, we see they are almost identical.

4. Where is $f'(x)$ positive? Notice this is where the graph of $f(x)$ is increasing. Where is $f'(x)$ negative? Notice this is where $f(x)$ is decreasing.

5. Next, find the minimum point of $f'(x)$ by first making sure the trace cursor is on the graph of the derivative, pressing $\boxed{\text{2ndF}}$ $\boxed{\text{CALC}}$, and double touching **3 Minimum**.

6. Look at $f(x)$ and observe that this appears to be the point at which the function "bends a different way."

7. Find the point of inflection directly by moving the cursor to the original function and pressing $\boxed{\text{2ndF}}$ $\boxed{\text{CALC}}$, touching ◄ on the screen, and double touching **7 Inflec**.

NAME ______________________ CLASS __________ DATE __________

4.3

GRAPHS OF DERIVATIVES

1. Graph the second derivative by pressing [Y=] [ENTER] [ENTER], and input *d*/*dx* (Y2) in the Y3 location with the keystrokes [MATH], touch **A CALC**, double touch **05 *d*/*dx*(** , press [VARS] [ENTER], touch **A XY**, double touch **2 Y2**, and press [)] [ENTER].

2. Press [GRAPH] to obtain the graphs of *f*(*x*), *f*'(*x*), and *f*''(*x*). Where is *f*''(*x*). zero? After pressing [TRACE] [▼] [▼] to place the tracer on the second derivative, press [2ndF] [CALC] and double touch **5 X_Incpt** to find that *f*''(*x*). = 0 at X= 1.167.

3. Find the *y*-value of this point of inflection by pressing [▲] [▲] to move the cursor to the original function. Press [2ndF] [CALC], double touch **1 Value**, and enter 1.167.

4. The connections we have discovered between the graphs of *f*(*x*), *f*'(*x*), and *f*''(*x*) are summarized in the tables below.

Interval	$f(x)$ is	$f'(x)$ is	$f''(x)$ is
$x <$ _____	increasing, concave down	positive	negative
_____ $< x <$ _____	decreasing, concave down	negative	negative
_____ $< x <$ _____	decreasing, concave up	negative	positive
$x >$ _____	increasing, concave up	positive	positive

x-value of point	$f(x)$ has	$f'(x)$ has	$f''(x)$ has
$x =$ _____	relative maximum	x-intercept	---
$x =$ _____	inflection point	minimum	x-intercept
$x =$ _____	relative minimum	x-intercept	---

NAME ______________________ CLASS __________ DATE __________

4.4

GRAPHS OF DERIVATIVES

In each activity below, press Y= and enter *f*(*x*) in Y1, *d*/*dx* (Y1) in Y2, and *d*/*dx* (Y2) in Y3.

Activity 1

1. Choose the dot graphing mode by pressing [2ndF] [FORMAT], touch **E STYLE1**, and double touch **2 Dot**. Press [2ndF] [QUIT] to exit the the FORMAT menu. Press [ZOOM], touch ▼ on the screen, and touch **7 Dec** to set the Decimal viewing window.

2. Use the graphs of $f(x)$, $f'(x)$, and $f''(x)$ to fill in the information in the tables below for the function $f(x) = |x+1|$.

Interval	$f(x)$ is	$f'(x)$ is	$f''(x)$ is
$x < 0$			
$x > 0$			

x-value of point	$f(x)$ has	$f'(x)$	$f''(x)$
$x = -1$			

Activity 2

1. Continue with the dot graphing mode and Decimal viewing window. Use the graphs of $f(x)$, $f'(x)$, and $f''(x)$ to fill in the information in the tables below for the function $f(x) = \sqrt{(9 - x^2)}$.

Interval	$f(x)$ is	$f'(x)$ is	$f''(x)$ is
$-3 < x < 0$			
$0 < x < 3$			

x-value of point	$f(x)$ has	$f'(x)$ has	$f''(x)$ has
$x = 0$			

NAME ____ CLASS __________ DATE __________

5.1

OPTIMIZATION

1. Set the calculator to radian measure by pressing 2ndF SET UP , touching **B DRG**, and double touching **2 Rad**. Set the calculator to floating point decimal display by touching **C FSE** and double touch **1 FloatPt**. Set the calculator to rectangular graphing by touching **E COORD**, and double touching **1 Rect**. Press 2ndF QUIT to exit the SET UP menu.

2. A new product was introduced through a television advertisement appearing during the Super Bowl. Suppose that the proportion of people that purchased the product x days after the advertisement appeared is given by $f(x) = \dfrac{(5.3x)}{(x^2 + 15)}$. When did maximum sales occur and what proportion of people purchased the product at that time?

3. Press Y= CL to clear the Y1 prompt. Press ENTER CL to clear additional prompts. Enter $f(x)$ in the Y1 location with the keystrokes a/b 5 × 3 X/θ/T/n ▶ X/θ/T/n x^2 + 1 5 .

4. Let's examine the graph for the first 25 days after the advertisement appeared. Press WINDOW , enter Xmin = 0, Xmax = 25, Xscl = 5, Ymin = 0, Ymax = 1, Yscl = 1.

5. Press GRAPH to view the graph.

6. When did maximum sales occur and what proportion of people purchased the product at that time? Press 2ndF CALC and double touch **4 Maximum**.

7. Is it the absolute maximum? Press WINDOW and change Xmax to 100. Press GRAPH to view the graph of the function. Repeat the procedure for Xmax= 500.

8. The maximum sales occurred __________ days after the advertisement first appeared, and the proportion of people that purchased the product at that time is __________ .

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5.2

OPTIMIZATION

1. A metal container with no top in the form of a right circular cylinder is being designed to hold 183 in^3 of liquid. If the material for the container costs 14¢ per square inch and the cost of welding the seams around the circular bottom and up the side cost 5¢ per inch, find the radius of the container with the smallest cost. What is the minimum cost?

2. The area of the container is equal to the area of lateral surface + area of bottom ($2\pi rh + \pi r^2$). The length of welds is equal to $h + 2\pi R$. The cost of the container is equal to .14(area of container) + .05(length of welds). Therefore, cost is equal to $.14(2\pi rh + \pi r^2) + .05(h + 2\pi r)$. We know that $183 = \pi r^2 h$, so $h = \dfrac{183}{(\pi r^2)}$.

3. Substituting this in the cost equation, we find cost of the container is $.14(2\pi r(\dfrac{183}{(\pi r^2)}) + \pi r^2) + .05(\dfrac{183}{(\pi r^2)}) + 2\pi r)$.

4. Press $\boxed{\text{Y=}}$ and clear any previously-entered functions with $\boxed{\text{CL}}$. Enter the simplified expression $\dfrac{51.24}{x} + .14\pi x^2 + \dfrac{9.15}{(\pi x^2)} + .1\pi x$, in Y1 with the keystrokes:
   $\boxed{\text{a/b}}$ $\boxed{5}$ $\boxed{1}$ $\boxed{\bullet}$ $\boxed{2}$ $\boxed{4}$ $\boxed{\blacktriangleright}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{\blacktriangleright}$ $\boxed{+}$ $\boxed{\bullet}$ $\boxed{1}$ $\boxed{4}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{x^2}$ $\boxed{+}$ $\boxed{\text{a/b}}$ $\boxed{9}$ $\boxed{\bullet}$ $\boxed{1}$ $\boxed{5}$ $\boxed{\blacktriangleright}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{x^2}$ $\boxed{\blacktriangleright}$ $\boxed{+}$ $\boxed{\bullet}$ $\boxed{1}$ $\boxed{\text{2ndF}}$ $\boxed{\pi}$ $\boxed{\text{X/}\theta\text{/T/}n}$.

5. Set the viewing window to $0 < x < 10$ and $0 < y < 50$. Enter the viewing window and press $\boxed{\text{GRAPH}}$ to view the graph of the cost function.

6. To find the minimum cost, press $\boxed{\text{2ndF}}$ $\boxed{\text{CALC}}$ and double touch **3 Minimum**. The radius of the container with the smallest cost is __________ inches. The minimum cost of the container is $__________.

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5.3

OPTIMIZATION

In each activity, choose a viewing window appropriate for the problem situation. Use the graph of the function to find the optimal solution.

Activity 1

A rumor about a new dress code spreads through a school. The proportion of students who have heard the rumor at time t days since the rumor started is given by $p(t) = \frac{6.4t}{(t^2 + 12)}$.

1. Because t is the number of days since the rumor started, let $t \geq$ ____.
   Because $p(t)$ is a proportion, let ____ $\leq p(t) \leq$ ____.

2. Graph the proportion of students who heard the rumor.
   Indicate your viewing window in the blanks provided:
   Xmin = ____, Xmax = ____, Xscl = ____,
   Ymin = ____, Ymax = ____, Yscl = ____.

3. What is the largest proportion of students who heard the rumor? _____

4. How many days after the rumor started had the largest proportion heard? _____

Activity 2

Lee is going to put a fence around part of the back yard. In order to have room for a tool shed and the family dog, a rectangular region whose area is approximately 675 square feet needs to be fenced. The back of the house is to be used as one side of the fenced-in portion.

1. Sketch the region to be fenced, indicating what your variable(s) represent. Write an equation representing the amount of fence Lee will buy.
   ____

2. What dimensions will minimize the amount of fence Lee will buy?
   ________________feet by________________feet

NAME ______________________ CLASS __________ DATE __________

6.1

SHADING AND CALCULATING AREAS REPRESENTED BY AN INTEGRAL

1. Set the calculator to radian measure by pressing [2ndF] [SET UP], touching **B DRG**, and double touching **2 Rad**. Set the calculator to floating point decimal display by touching **C FSE** and double touch **1 FloatPt**. Set the calculator to rectangular graphing by touching **E COORD**, and double touching **1 Rect**. Press [2ndF] [QUIT] to exit the SET UP menu.

2. Find an estimate of $\int_0^1 2x\,dx$.

3. Integrate a function by pressing [$\boxplus\boxminus\boxtimes\boxdiv$] [MATH], touch **A CALC**, and double touch **06** $\int$.

4. Enter 0 for the lower limit. Press [▲], input 1 for the upper limit, and press [▶]. Next, press [2] [X/θ/T/*n*] to input the integrand. Enter the "dx" by pressing [MATH] and double touching **07 dx**. Press [ENTER] to compute.

5. Shade the region by first pressing [Y=] [CL] to access and clear the Y1 prompt. Clear additional prompts by pressing [ENTER] [CL].

6. Enter $f(x)$ in Y1 with the keystrokes [2] [X/θ/T/*n*]. Press [WINDOW] and enter Xmin = 0 and Xmax = 1. Draw the graph by pressing [ZOOM], touching **A ZOOM**, and double touching **1 Auto**.

7. Shade the region by first pressing [2ndF] [DRAW], touch **G SHADE**, and double touch **1 Set** to access the shading screen.

8. Since Y1= 2X is the function "on the top," press [▶], and touch Y1 on the screen. Since we are only dealing with one function, leave the lower bound location empty. Press [GRAPH] to view the shaded region.

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6.2

SHADING AND CALCULATING AREAS REPRESENTED BY AN INTEGRAL

1. Calculate and draw a graph of the area of the region between $f(x) = 5x - x^2 + 12$ and $g(x) = e^x + 5$.

2. First, press [2ndF] [DRAW], touch **G SHADE**, and double touch **2 INITIAL**. Return to and clear the Y prompts by pressing [Y=] [CL]. Clear additional prompts if necessary.

3. Input $f(x)$ in Y1 with the keystrokes [5] [X/θ/T/*n*] [–] [X/θ/T/*n*] [x^2] [+] [1] [2] [ENTER]. Input $g(x)$ in Y2 with the keystrokes [2ndF] [e^x] [X/θ/T/*n*] [▶] [+] [5].

4. Enter the viewing window $^-6.3 < x < 6.3$ and $^-10 < y < 30$. Your viewing window should clearly show the region between $f(x)$ and $g(x)$ and, if applicable, display the intersections of the functions. Press [GRAPH] to view the graphs.

5. Shade the region between the two curves by pressing [2ndF] [DRAW], touch **G SHADE**, and double touch **1 SET**. Touch Y2 since Y2 is the function "on the bottom," pressing [▶], and touch Y1 since Y1 is the function "on the top." Press [GRAPH] to view the shaded region.

6. Next, find the limits of integration. Press [TRACE] [2ndF] [CALC] and double touch **2 Intsct**. Do this twice to obtain the *x*-coordinates of the two points of intersection.

7. Press [⊞⊟⊠⊡] [CL], press [MATH], touch **A CALC**, double touch 06 ∫, enter the lower bound, press [▲], enter the upper bound, press [▶], enter the function "on the top," $5x - x^2 + 12$, press [–] [(], enter the function "on the bottom," $e^x + 5$, press [)], press [MATH] and double touch **07 dx**. Press [ENTER] to obtain the area.

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6.3

SHADING AND CALCULATING AREAS REPRESENTED BY AN INTEGRAL

The *average value* of a continuous function $y = f(x)$ over the interval from $x = a$ to $x = b$ is given by $\left(\frac{1}{(b-a)}\right) \int_a^b f(x)\,dx$.

Activity 1

1. Use the calculator's numeric integration function to approximate $\int_{-4}^{0} (x+2)^2\,dx$. ______________

2. Graph the region whose area this integral represents. Shade the region. Sketch the shaded region below.

3. Find the average value of $f(x) = (x+2)^2$ over the interval $^-4 \le x \le 0$. _________

Activity 2

The temperature at time t, measured in hours since midnight over a 24-hour period on a certain day in June at Newark airport, is approximated by the function $f(t) = 80 + 10\sin(.26t - 2.3)$ degrees Fahrenheit.

1. Enter $f(t)$ in Y1. Construct an autoscaled graph over the interval $0 \le t \le 24$.

2. Find the minimum and maximum temperatures over a 24-hour period and the approximate hour t which they occur.
   minimum=______°F at______, maximum=______°F at______.

3. Find the average temperature over the 24-hour period.______°F

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7.1

PROGRAM FOR RECTANGULAR APPROXIMATION OF AREA

Set up and enter a program:

1. Turn the calculator on and set to radian mode by pressing [2ndF] [SET UP], touch **B DRG**, double touch **2 Rad**. Press [2ndF] [QUIT] to exit the SET UP menu.

2. Press [2ndF] [PRGM] to enter programming mode. Touch **C NEW** to enter a new program, followed by [ENTER]. Name the new program RECTAPP by pressing [R] [E] [C] [T] [A] [P] [P] followed by [ENTER].

3. You can now enter in the RECTAPP program. Remember to press [ENTER] at the end of each line. If you make a mistake, use the calculator's editing features to make corrections. Enter the following program:

PROGRAM	KEYSTROKES
Input N	[2ndF] [PRGM] [A] [3] [ALPHA] [N] [ENTER]
Input A	[2ndF] [PRGM] [3] [ALPHA] [A] [ENTER]
Y1(A)⇒L	[VARS] [A] [ENTER] [A] [1] [(] [ALPHA] [A] [)] [STO] [ALPHA] [L] [ENTER]
Input B	[2ndF] [PRGM] [3] [ALPHA] [B] [ENTER]
Y1(B)⇒R	[VARS] [ENTER] [1] [(] [ALPHA] [B] [)] [STO] [ALPHA] [R] [ENTER]
(B–A)/N⇒W	[(] [ALPHA] [B] [–] [ALPHA] [A] [)] [÷] [ALPHA] [N] [STO] [ALPHA] [W] [ENTER]
A+W/2⇒X	[ALPHA] [A] [+] [ALPHA] [W] [÷] [2] [STO] [X/θ/T/*n*] [ENTER]

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7.2

PROGRAM FOR RECTANGULAR APPROXIMATION OF AREA

Entering a program (cont.)

Y1(X)⇒M	VARS ENTER 1 (X/θ/T/*n*) STO ALPHA M ENTER
A+W⇒X	ALPHA A + ALPHA W STO X/θ/T/*n* ENTER
Label LOOP	2ndF PRGM B 1 2ndF A-LOCK L O O P ALPHA ENTER
Y1(X)+L⇒L	VARS ENTER 1 (X/θ/T/*n*) + ALPHA L STO ALPHA L ENTER
Y1(X)+R⇒R	VARS ENTER 1 (X/θ/T/*n*) + ALPHA R STO ALPHA R ENTER
X+W/2⇒X	X/θ/T/*n* + ALPHA W ÷ 2 STO X/θ/T/*n* ENTER
Y1(X)+M⇒M	VARS ENTER 1 (X/θ/T/*n*) + ALPHA M STO ALPHA M ENTER
X+W/2⇒X	X/θ/T/*n* + ALPHA W ÷ 2 STO X/θ/T/*n* ENTER
If X<BGoto	2ndF PRGM 3 X/θ/T/*n*
LOOP	MATH F 5 ALPHA B 2ndF PRGM 2 2ndF A-LOCK L O O P ENTER
ClrT	2ndF PRGM C 1 ENTER
Print "LEFT="	2ndF PRGM A 1 2ndF PRGM 2 2ndF A-LOCK L E F T = 2ndF PRGM 2 ENTER
Print W•L	2ndF PRGM 1 ALPHA W × ALPHA L ENTER

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7.3

PROGRAMS FOR RECTANGULAR AND TRAPEZOIDAL APPROXIMATION OF AREA

Entering a program (cont.)

Print "MID="	2ndF PRGM 1 2ndF PRGM 2 2ndF A-LOCK M I D = 2ndF PRGM 2 ENTER
Print W*M	2ndF PRGM 1 ALPHA W × ALPHA M ENTER
Print "RIGHT="	2ndF PRGM 1 2ndF PRGM 2 2ndF A-LOCK R I G H T = 2ndF PRGM 2 ENTER
Print W*R	2ndF PRGM 1 ALPHA W × ALPHA R ENTER
End	2ndF PRGM 6 ENTER

Press 2ndF QUIT to save the program and exit the editing mode.

Executing a program

Enter the function you wish to approximate, say $f(x) = \cos x$, by pressing Y= CL cos X/θ/T/*n* . Press ENTER CL to clear additional prompts. To execute the RECTAPP program, press 2ndF PRGM A , highlight the 'RECTAPP' program, and press ENTER . Enter 'N' of 5 by pressing 5 ENTER , followed by 'A' of 0 and 'B' of 1. You will see a display of left, midpoint and right rectangular approximations for the area under the cosine function between 0 and 1 with 5 intervals in the partition.

TRAPAPP program.

PROGRAM	KEYSTROKES
Input N	2ndF PRGM A 3 ALPHA N ENTER

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7.4

TRAPEZOIDAL APPROXIMATION OF AREA

Input A	$\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{3}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{\text{ENTER}}$
Y1(A)⇒L	$\boxed{\text{VARS}}$ $\boxed{\text{A}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{A}}$ $\boxed{1}$ $\boxed{(}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{)}$ $\boxed{\text{STO}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{L}}$ $\boxed{\text{ENTER}}$
Input B	$\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{3}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$ $\boxed{\text{ENTER}}$
Y1(B)⇒R	$\boxed{\text{VARS}}$ $\boxed{\text{ENTER}}$ $\boxed{1}$ $\boxed{(}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$ $\boxed{)}$ $\boxed{\text{STO}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{R}}$ $\boxed{\text{ENTER}}$
0→Z	$\boxed{0}$ $\boxed{\text{STO}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{Z}}$ $\boxed{\text{ENTER}}$
(B–A)/N⇒W	$\boxed{(}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$ $\boxed{-}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{)}$ $\boxed{\div}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{N}}$ $\boxed{\text{STO}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{W}}$ $\boxed{\text{ENTER}}$
A+W⇒X	$\boxed{\text{ALPHA}}$ $\boxed{\text{A}}$ $\boxed{+}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{W}}$ $\boxed{\text{STO}}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{\text{ENTER}}$
Label LOOP	$\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{\text{B}}$ $\boxed{1}$ $\boxed{\text{2ndF}}$ $\boxed{\text{A-LOCK}}$ $\boxed{\text{L}}$ $\boxed{\text{O}}$ $\boxed{\text{O}}$ $\boxed{\text{P}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{ENTER}}$
2Y1(X)+Z⇒Z	$\boxed{2}$ $\boxed{\text{VARS}}$ $\boxed{\text{ENTER}}$ $\boxed{1}$ $\boxed{(}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{)}$ $\boxed{+}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{Z}}$ $\boxed{\text{STO}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{Z}}$ $\boxed{\text{ENTER}}$
X+W⇒X	$\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{+}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{W}}$ $\boxed{\text{STO}}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{\text{ENTER}}$
If X<BGoto	$\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{3}$ $\boxed{\text{X}/\theta/\text{T}/n}$
LOOP	$\boxed{\text{MATH}}$ $\boxed{\text{F}}$ $\boxed{5}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$ $\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{2}$ $\boxed{\text{2ndF}}$ $\boxed{\text{A-LOCK}}$ $\boxed{\text{L}}$ $\boxed{\text{O}}$ $\boxed{\text{O}}$ $\boxed{\text{P}}$ $\boxed{\text{ENTER}}$
ClrT	$\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{\text{C}}$ $\boxed{1}$ $\boxed{\text{ENTER}}$
Print "TRAP="	$\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{\text{A}}$ $\boxed{1}$ $\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{2}$ $\boxed{\text{2ndF}}$ $\boxed{\text{A-LOCK}}$ $\boxed{\text{T}}$ $\boxed{\text{R}}$ $\boxed{\text{A}}$ $\boxed{\text{P}}$ $\boxed{=}$ $\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{2}$ $\boxed{\text{ENTER}}$
Print (W/2)	$\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{1}$ $\boxed{(}$ $\boxed{\text{ALPHA}}$
(L+Z+R)	$\boxed{\text{W}}$ $\boxed{\div}$ $\boxed{2}$ $\boxed{)}$ $\boxed{(}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{L}}$ $\boxed{+}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{Z}}$ $\boxed{+}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{R}}$ $\boxed{)}$ $\boxed{\text{ENTER}}$
End	$\boxed{\text{2ndF}}$ $\boxed{\text{PRGM}}$ $\boxed{6}$ $\boxed{\text{ENTER}}$

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8.1

HYPERBOLIC FUNCTIONS

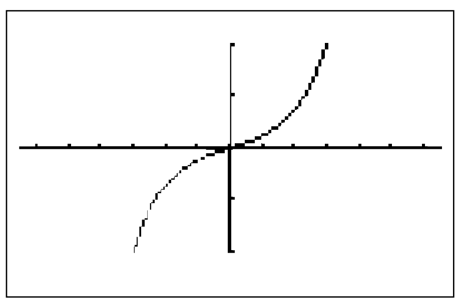
Steps for graphing the hyperbolic sine function.

1. Turn the calculator on and press [Y=] [CL] to access and clear the [Y1] prompt. Press [ENTER] [CL] to remove additional expressions.

2. The calculator should be setup in radian measurement with rectangular coordinates. To complete this setup, press [2ndF] [SET UP] , touch **B DRG**, double touch **2 Rad**, touch **E COORD**, and double touch **1 Rect**. Press [2nd] [QUIT] to exit the SET UP menu and return to the Y prompts.

3. To enter the hyperbolic sine function (y = sinh x) for Y1, press [MATH], touch **A CALC**, touch ▼ on the screen, double touch **15 sinh**, and press [X/θ/T/*n*] .

4. Enter the viewing window by pressing [ZOOM], touching **F HYP**, and double touching **1 sinh *x***.



Steps for graphing the hyperbolic cosine function.

1. Press [Y=] [CL] to remove the hyperbolic sine function.

2. To enter the hyperbolic cosine function (y = cosh x) for Y1, press [MATH] , touch ▼ on the screen, double touch **16 cosh *x***, press [X/θ/T/*n*] .

3. Use the same viewing window as before ($^-6.5 \le x \le 6.5$ by $^-10 \le y \le 10$) and press [GRAPH] to view the graph.

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8.2

HYPERBOLIC FUNCTIONS

Activity 1 Graph the hyperbolic tangent function.

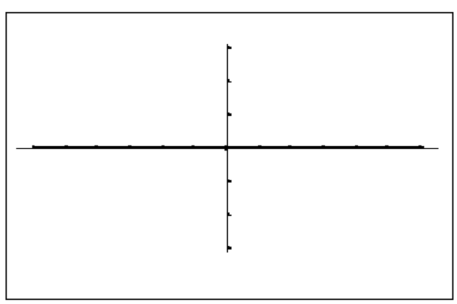
1. Turn the calculator on and press Y= CL to access and clear the Y1 prompt. Press ENTER CL to remove additional expressions.

2. The calculator should be setup in radian measurement with rectangular coordinates. To complete this setup, press 2ndF SET UP, touch **B DRG**, double touch **2 Rad**, touch **E COORD**, and double touch **1 Rect**. Press 2nd QUIT to exit the SET UP menu and return to the Y prompts.

3. To enter the hyperbolic tangent function ($y = \tanh x$) for Y1, press MATH, touch A CALC, touch ▼ on the screen, double touch **17 tanh**, and press X/θ/T/*n* .

4. Enter the viewing window by pressing ZOOM, touching **F HYP**, and double touching **3 tanh *x***.

5. Sketch the graph in the box provided.



Activity 2 Graph the hyperbolic cotangent function.

Repeat the steps for Activity 1 for the hyperbolic cotangent function. Enter the hyperbolic cotangent function ($y = \coth x$) as $y = 1/\tanh x$. Continue to use the tanh viewing window.

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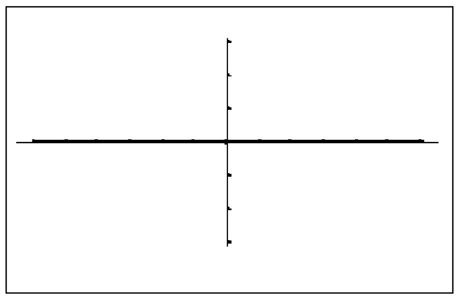
8.3

HYPERBOLIC FUNCTIONS

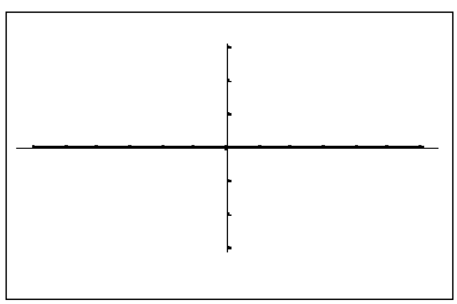
Activity 3 Graph the hyperbolic secant function.

Repeat the steps for Activity 1 for the hyperbolic secant function. Enter the hyperbolic secant function (y = sech x) as y = 1/cosh x. Continue to use the tanh viewing window.



Activity 4 Graph the hyperbolic cosecant function.

Repeat the steps for Activity 1 for the hyperbolic cosecant function. Enter the hyperbolic cosecant function (y = csch x) as y = 1/sinh x. Continue to use the tanh viewing window.



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9.1

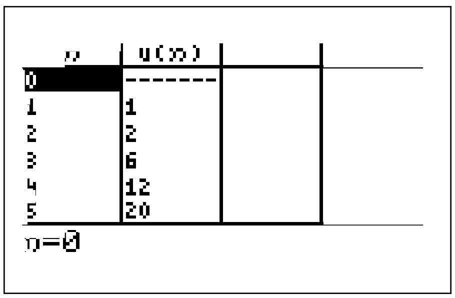
SEQUENCES

1. Turn the calculator on and set the calculator to sequence mode by pressing [2ndF] [SET UP], touching **E COORD**, and double touching **4 Seq**.

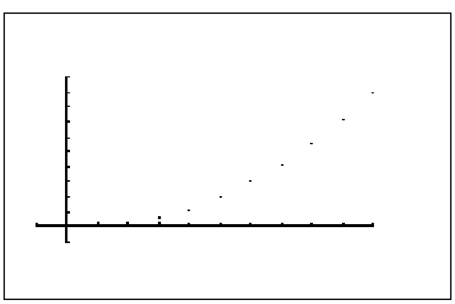
2. Press [2ndF] [QUIT] [Y=] to access the sequence prompts. Clear any sequences by presing [CL] .

3. Enter the sequence generator $a_n = n^2 - n$ for $u(n)$ by pressing [X/θ/T/n] [x^2] [–] [X/θ/T/n] [ENTER]. Enter $n_1 = 1$ for $u(n\text{Min})$ by pressing [1] [ENTER].

4. View a table of sequence values by pressing [TABLE].



5. Graph the sequence by first setting the format to time and dot modes. Do this by pressing [2ndF] [FORMAT] ,touching **G TYPE**, double touching **2 Time**, touching **E STYLE1**, and double touching **2 Dot**. Press [2ndF] [QUIT] to exit the FORMAT menu. Enter the viewing window by pressing [WINDOW] [EZ], double touching **3**, double touching **5 -1<X<10**, and double touching **2 -10<Y<100**.



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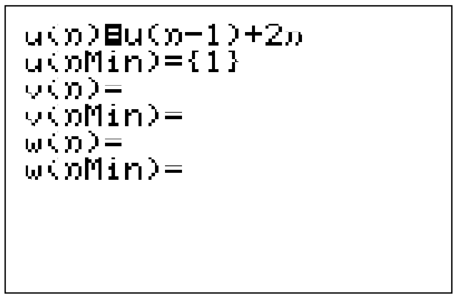
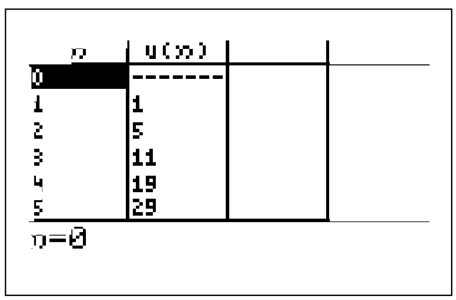
9.2

SEQUENCES

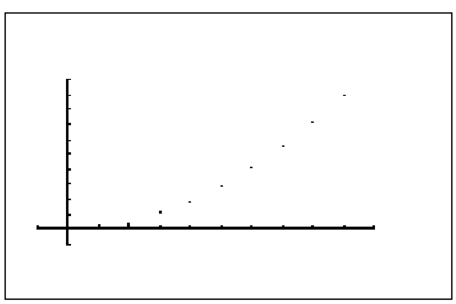
1. Enter the recursive sequence generator $a_n = a_{n-1} + 2n$ for $u(n)$ by pressing [Y=] [CL] [2ndF] [u] [(] [X/θ/T/n] [–] [1] [)] [+] [2] [X/θ/T/n] [ENTER]. Enter $a_1 = 1$ by pressing [1] [ENTER].



2. View a table of sequence values by pressing [TABLE].



3. Graph the sequence in the same window as 9.1 by pressing [GRAPH].



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9.3

SEQUENCES

Activity 1

The Fibonnoci sequence is the sequence where the previous two terms are added together to form the next term. The first two terms are $a_1 = 1$ and $a_2 = 1$.

1. Enter the Fibonnoci sequence generator $a_n = a_{n-1} + a_{n-2}$ for $u(n)$ by pressing $\boxed{\text{Y=}}$ $\boxed{\text{CL}}$ $\boxed{\text{2ndF}}$ $\boxed{\text{u}}$ $\boxed{(}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{-}$ $\boxed{1}$ $\boxed{)}$ $\boxed{+}$ $\boxed{\text{2ndF}}$ $\boxed{\text{u}}$ $\boxed{(}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{-}$ $\boxed{2}$ $\boxed{)}$ $\boxed{\text{ENTER}}$. Enter $a_1 = 1$ and $a_2 = 1$ by pressing $\boxed{\text{2ndF}}$ $\boxed{\{}$ $\boxed{1}$ $\boxed{,}$ $\boxed{1}$ $\boxed{\text{2ndF}}$ $\boxed{\}}$ $\boxed{\text{ENTER}}$.

2. View a table of sequence values by pressing $\boxed{\text{TABLE}}$.

3. Graph the sequence in the same window as 9.1 by pressing $\boxed{\text{GRAPH}}$.

Activity 2

Find the partial sum of a series, $\sum$ 1/X, for the first 10 terms.

1. Return the SET UP to rectangular mode by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touching **E COORD**, and double touching **1 Rect**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the SET UP menu.

2. First find the sequence of the first 10 terms of series $\sum$ 1/X by first pressing $\boxed{\begin{smallmatrix}+ & - \\ \times & \div\end{smallmatrix}}$ $\boxed{\text{CL}}$ $\boxed{\text{2ndF}}$ $\boxed{\text{LIST}}$, touching **A OPE**, and double touching **5 seq(** . Enter the generator 1/X by pressing $\boxed{1}$ $\boxed{\div}$ $\boxed{\text{X/}\theta\text{/T/}n}$ $\boxed{,}$. Enter the lower and upper bounds for the sequence by pressing $\boxed{1}$ $\boxed{,}$ $\boxed{1}$ $\boxed{0}$ $\boxed{)}$. Find the sequence by pressing $\boxed{\text{ENTER}}$. Press $\boxed{\blacktriangleright}$ to see more of the sequence.

3. Find the partial sum of the first 10 terms by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{LIST}}$, double touching **6 cumul**, and pressing $\boxed{\text{2ndF}}$ $\boxed{\text{ANS}}$ $\boxed{\text{ENTER}}$. Press $\boxed{\blacktriangleright}$ to move right in the sequence of partial sums until the last term is seen. This is the partial sum of the first 10 terms.

NAME ____________________ CLASS __________ DATE __________

10.1

GRAPHING POLAR EQUATIONS

Steps for graphing a polar function:

1. Turn the calculator on and press 2ndF SET UP, touch **E COORD**, and double touch **3 Polar** to change to polar mode. While in the SET UP menu, the calculator should be setup in radian mode. To complete this setup, touch **B DRG**, and double touch **2 Rad**. Press 2ndF QUIT to exit the SET UP menu.

2. Make sure calculator is in connected mode by pressing 2ndF FORMAT, touching **E STYLE1**, and double touching **1 Connect**. While in the FORMAT menu, set the calculator to display polar coordinates when tracing, by touching **B CURSOR** and double touching **2 PolarCoord**. Also, set the calculator to display the expression during tracing by touching **C EXPRESS** and double touching **1 ON**. Press 2ndF QUIT to exit the FORMAT menu.

3. Press Y= to access the R1 prompt. Press CL to remove an old R1 expression. Press ENTER CL to clear additional R prompts.

4. Enter the polar function $r = 2(1 - \cos \theta)$ for R1, by pressing 2 (1 – cos X/θ/T/*n*). Notice, when in polar mode the X/θ/T/*n* key provides a θ for equation entry.

5. Now, graph the polar function in the Decimal viewing window by pressing ZOOM, touching **A ZOOM**, touching ▼ on the screen, and touching **7 Dec.**

6. This particular shape of curve is called a cardoid. Trace the curve by pressing TRACE. Notice the expression is displayed at the top of the screen.

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10.2

GRAPHING PARAMETRIC EQUATIONS

Steps for graphing a parametric function:

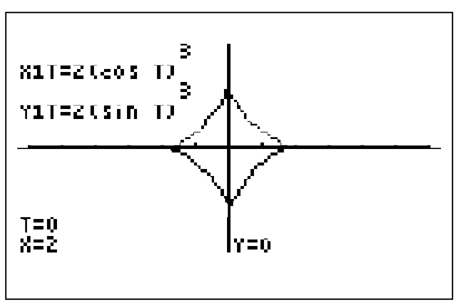
1. Turn the calculator on and press $\boxed{\text{2ndF}}$ $\boxed{\text{SET UP}}$, touch **E COORD**, and double touch **2 Param** to change to parametric mode. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the SET UP menu.

2. Make sure calculator is set to display rectangular coordinates when tracing by pressing $\boxed{\text{2ndF}}$ $\boxed{\text{FORMAT}}$, touching **B CURSOR** and double touching **1 RectCoord**. Press $\boxed{\text{2ndF}}$ $\boxed{\text{QUIT}}$ to exit the FORMAT menu.

3. To enter the parametric function X1T = $2(\cos T)^3$, Y1T = $2(\sin T)^3$, press $\boxed{\text{Y=}}$ $\boxed{\text{CL}}$ $\boxed{2}$ $\boxed{(}$ $\boxed{\cos}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{)}$ $\boxed{a^b}$ $\boxed{3}$ $\boxed{\text{ENTER}}$ $\boxed{\text{CL}}$ $\boxed{2}$ $\boxed{(}$ $\boxed{\sin}$ $\boxed{\text{X}/\theta/\text{T}/n}$ $\boxed{)}$ $\boxed{a^b}$ $\boxed{3}$ $\boxed{\text{ENTER}}$. Notice, when in parametric mode the $\boxed{\text{X}/\theta/\text{T}/n}$ key provides a T for equation entry.

4. Now, graph the parametric function in the Decimal viewing window by pressing $\boxed{\text{ZOOM}}$, touching **A ZOOM**, touching ▼ on the screen, and touching **7 Dec**.

5. Press $\boxed{\text{TRACE}}$ and notice the expression and T values now appear on the range screen.



NAME ______________________________ CLASS __________ DATE __________

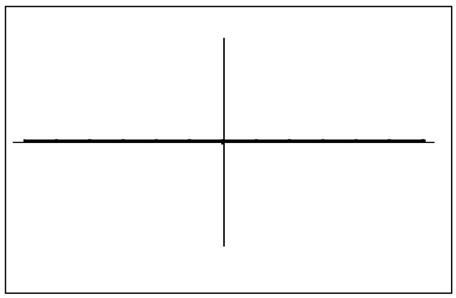
10.3

GRAPHING POLAR EQUATIONS

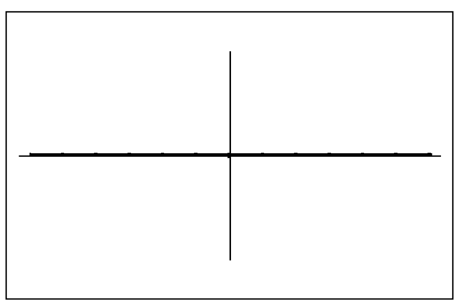
Activity 1

Graph the following polar functions. Sketch what you see in the box provided.

1. $r = 1 + 2 \cos \theta$ (limacon)
   Decimal viewing window.



2. $r = (4\sin(2\theta))$ (leminscate)
   Decimal viewing window.



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10.4

GRAPHING PARAMETRIC EQUATIONS

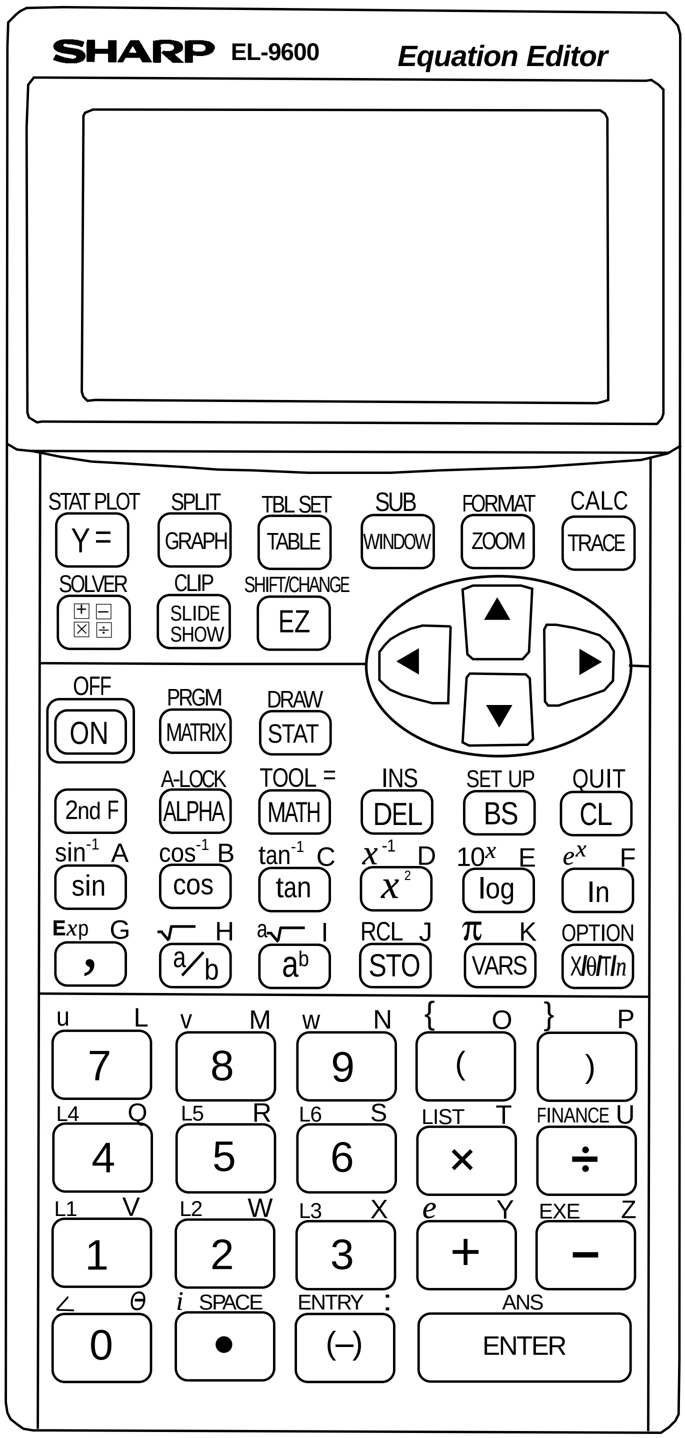
Activity 2

Graph the following parametric functions.

1. X1T = T – sin T (cycloid)
   Y1T = 1 – cos T
   Decimal viewing window

2. X1T = 2 cos T (ellipse)
   Y1T = sin T
   Decimal viewing window

KEYPAD FOR
THE SHARP EL-9600 GRAPHING CALCULATOR



SOLUTIONS TO SELECTED ACTIVITIES

1. EVALUATING LIMITS

ADDITIONAL PROBLEMS

1. 12; continuous for all values of x except $x = -2$
2. does not exist; continuous for all values of x except $x = 2.5$
3. e and 1; continuous for all values of $x>-1$ except $x=0$
4. 4.6; continuous for all values of x except $x = 500$
5. 71 wpm; No, 95

BLACKLINE MASTER 1.3

1. $f(x) = -1$, $f(x) = 1$, Yes
2. 1, 2/3, does not exist, 1
3. $p(20)=9082$, 10000

2. DERIVATIVES

ADDITIONAL PROBLEMS

1. 2
2. does not exist
3. 2
4. 3.14159
5. $-e^{-x}$, x, $1/x$

BLACKLINE MASTER 2.1

ACTIVITY 2

2. 6

ACTIVITY 3

1. 54

BLACKLINE MASTER 2.2

ACTIVITY 2

2. 6

ACTIVITY 3

1. does not exist

BLACKLINE MASTER

2.3 ACTIVITY 2

2. $2e^{2x}$, $5x$

3. TANGENT LINES

ADDITIONAL PROBLEMS

1. $y = 4x - 4$
2. $y = .54x + .301$
3. $y = 1.298x + .103$
4. $y = .5x + .5$
5. $y = 1$

4. GRAPHS OF DERIVATIVES

ADDITIONAL PROBLEMS

1. $x < .361$ dec, concave up relative min at $x = -.361$
   $-.361<x<1.667$ inc, concave up inflection point at $x = 1.667$
   $1.667<x<3.694$ inc, concave down relative max at $x = 3.694$
   $x>3.69$ dec, concave down
2. $x < -.707$ inc, concave up inflection point at $x = -.707$
   $-.707<x<0$ inc, concave down relative min at $x = 0$
   $0<x<.707$ dec, concave down inflection point at $x = .707$ $x>.707$ dec, concave up
3. $x>0$ inc, concave down no max or min
4. $x<0$ dec, concave up relative min at $x=0$
   $x<0$ inc, concave up

BLACKLINE MASTER 4.4

ACTIVITY 1

2. decreasing, negative, 0
   increasing, positive, 0
   minimum, does not exist, does not exist

ACTIVITY 2

1. increasing, concave down, positive, negative
   decreasing, concave down, positive, negative
   maximum, x-intercept, maximum

5. OPTIMIZATION

ADDITIONAL PROBLEMS

1. 1352
2. 4000.562 m
3. 14.063 ft, .375 seconds
4. 1.132 v
5. $x \approx 2.678$, $y \approx 2.231$

BLACKLINE MASTER 5.1

8. 3.873, .684

BLACKLINE MASTER 5.2

6. 3.799, 21.23

BLACKLINE MASTER 5.3

ACTIVITY 1

1. 0, 0, 1
2. 0, 25, 5, 0, 1, 1
3. .924
4. 3.464 days

ACTIVITY 2

1. amount of fence = $2w + l = 2w + 675/w$
2. $w \approx 18.371$, $l \approx 36.74$

6. SHADING AND CALCULATING AREAS REPRESENTED BY AN INTEGRAL

ADDITIONAL PROBLEMS

1. 246.738
2. 7.021
3. .828
4. .402
5. 4

BLACKLINE MASTER 6.3

ACTIVITY 1

1. 5.333
3. 1.333

ACTIVITY 2

2. 70, 2.805
   90, 14.888
3. 80.051

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